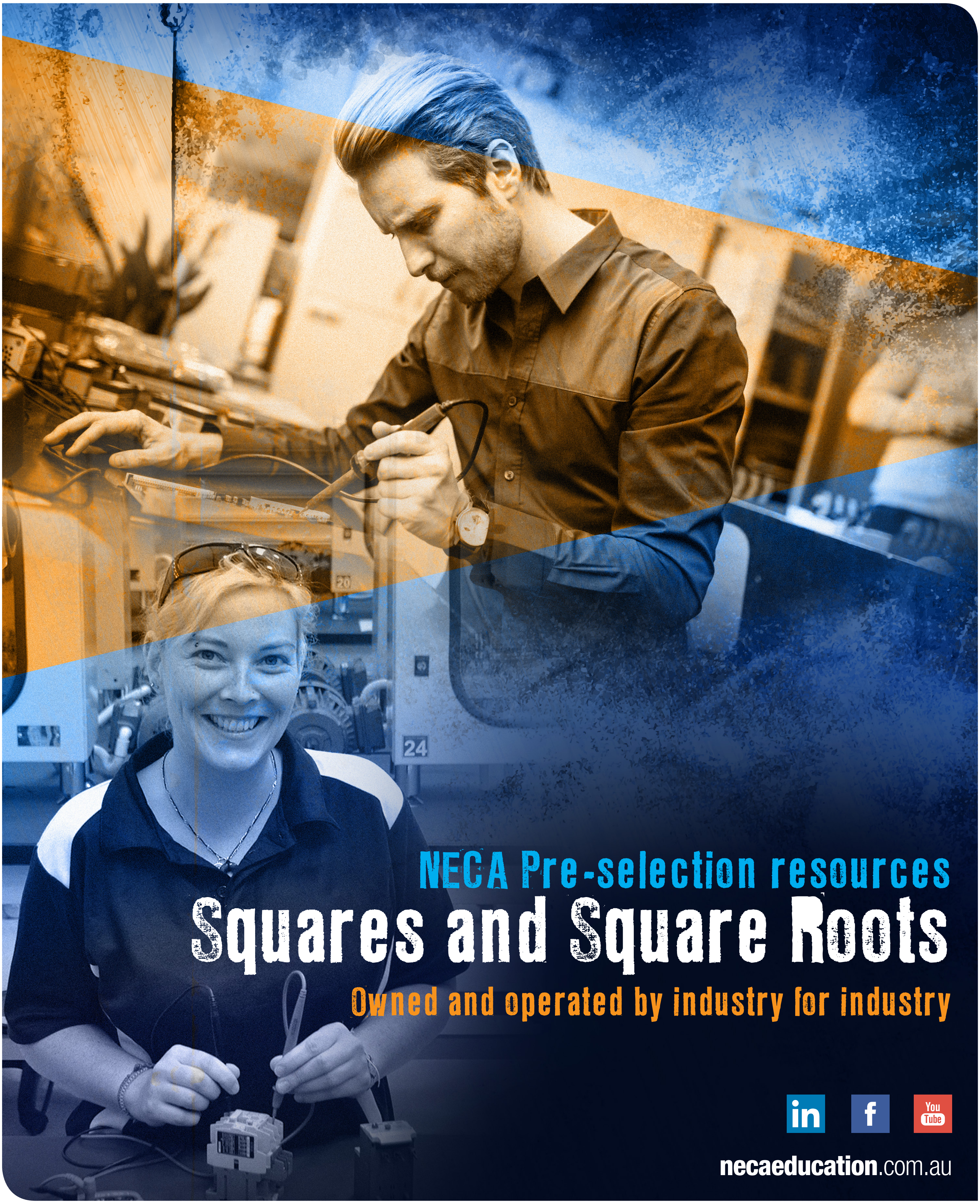

NECA Pre-selection resources

Squares and Square Roots

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SQUARES AND SQUARE ROOTS

In using a lot of mathematical formula in the Electrotechnology area we have to be able to square a number or find the square root of a number.

LEARNING OUTCOME

- Understand and perform calculations using squares and square roots.

PERFORMANCE CRITERIA

- Understand the concepts of squares and square roots.
- Identify the need for calculations involving squares and square roots.
- Use the calculator to make accurate work and make specific calculations.

SQUARES AND SQUARE ROOTS

In order to calculate areas and apply certain formulae such as Pythagoras' theorem, it is important to have an understanding of squares and square roots.

SQUARING A NUMBER

To square a number you must multiply a number the number by itself.

← **3** →

1	2	3
4	5	6
7	8	9

↑ **3** ↓

$3^2 = 9$

$3 \times 3 = 9$

3 **squared** equals 9

This is written as $3^2 = 9$

so	$4^2 = 16$	(4×4)	4 **squared** equals 16
	$5^2 = 25$	(5×5)	5 **squared** equals 25
	$8^2 = 64$	(8×8)	8 **squared** equals 64

EXERCISE 1

Calculate the following. Show your working out.

Example: $3^2 = 3 \times 3 = 9$

a. $10^2 =$

b. $9^2 =$

c. $12^2 =$

Using the calculator

To find 9^2

Method 1	or	Method 2

9 **x** **9** **=** **Answer 81**

9 **X^2** **Answer 81**

EXERCISE 2

Calculate the following, using the calculator.

a. $24^2 =$

b. $15^2 =$

c. $128^2 =$

d. $52^2 =$

☑ **Use the answer sheet to check your work.**

SQUARE ROOT

Finding the square root is the opposite of squaring a number.

$3^2 = 9$ (squaring)

$\sqrt{9} = 3$ (square root)

The square root of 9 is 3.

This is written as $\sqrt{9} = 3$

Now look at these:-

3 x 3 =9	or $3^2 = 9$	therefore	$\sqrt{9} = 3$
6 x 6 = 36	or $6^2 = 36$	therefore	$\sqrt{36} = 6$
16 x 16 = 256	or $16^2 = 256$	therefore	$\sqrt{256} = 16$

EXERCISE 3

Complete the following statements:

a.	4 x 4 =.....	or	4^2 = ...	therefore	$\sqrt{\ldots}$ = 4
b.	..... x..... = 81	or	.....^{2} = 81	therefore	$\sqrt{81}$ = ...
c.	..... x..... = 49	or	.....^{2} = 49	therefore	$\sqrt{49}$ = ...

Using the calculator

Note: Steps on some calculators may differ. Refer to your calculator guide.

$\sqrt{289}$ $\boxed{2}$ $\boxed{8}$ $\boxed{9}$ $\boxed{\sqrt{x}}$ Answer 17

Depression of the square root key tells the calculator to find the square root of the number on the display. The square root then appears on the display.

EXERCISE 4

Calculate the following (answers are provided to two decimal places)

a. $\sqrt{100}$ =

b. $\sqrt{529}$ =

c. $\sqrt{231}$ =

d. $\sqrt{54.76}$ =

e. $\sqrt{702.25}$ =

f. $\sqrt{5643.21}$ =

g. $\sqrt{5+8.2}$

5	+	8	.	2	=	$\sqrt{x}$	Answer

h. $\sqrt{17.9-2.6+32.8}$ =

I. $\sqrt{8}$ + $\sqrt{15}$ =

j. $\sqrt{7^2+24^2}$

☑ **Use the answer sheet to check your work.**

ANSWERS

EXERCISE 1

a.	$10^2 = 10 \times 10$	=	100
b.	$9^2 = 9 \times 9$	=	81
c.	$12^2 = 12 \times 12$	=	144

EXERCISE 2

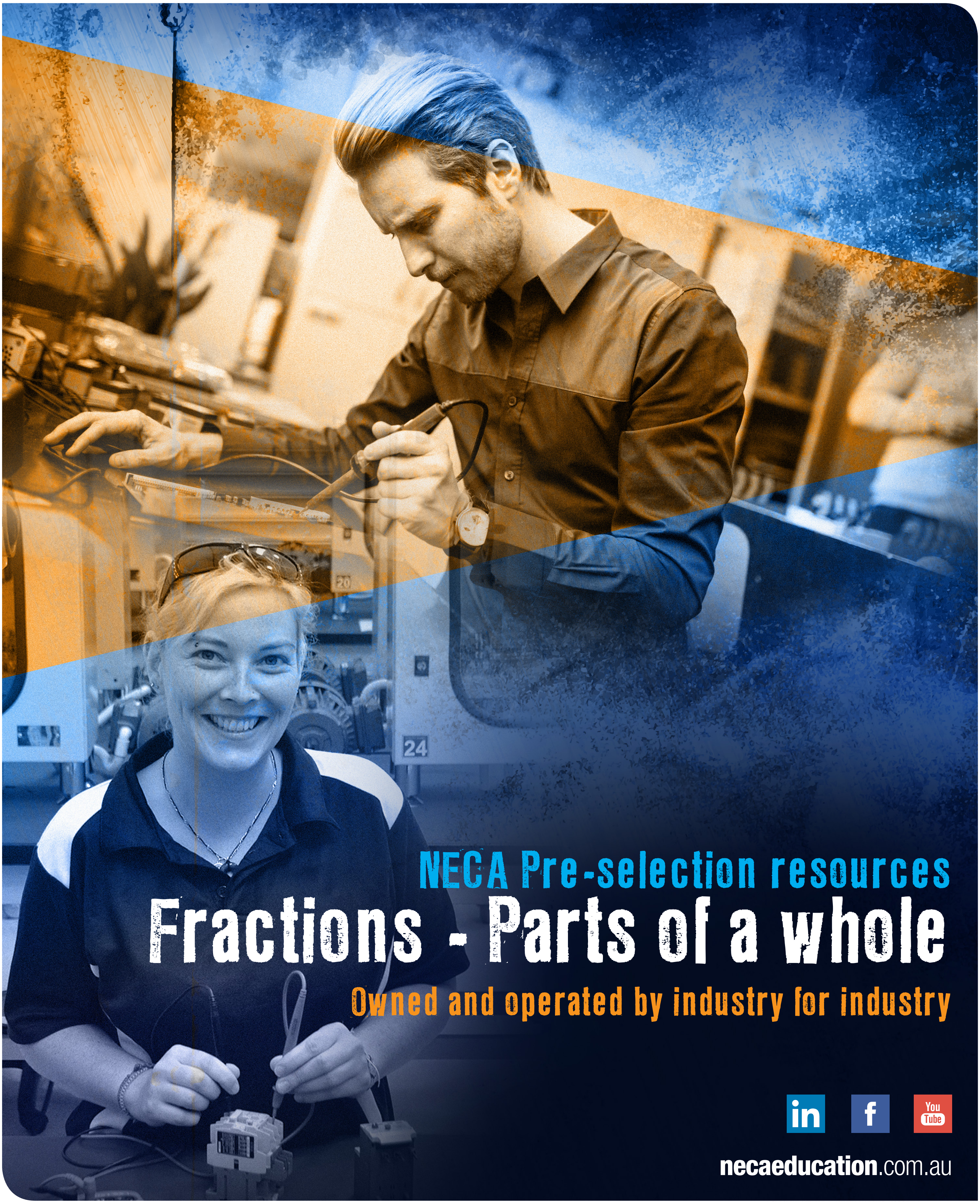
a.	24^2	=	576
b.	15^2	=	225
c.	128^2	=	16,384
d.	52^2	=	2,704

EXERCISE 3

a.	4 x 4 =	16	or	$4^2 = 16$	therefore $\sqrt{16} = 4$
b.	9 x 9 =	81	or	$9^2 = 81$	therefore $\sqrt{81} = 9$
c.	7 x 7 =	49	or	$7^2 = 49$	therefore $\sqrt{49} = 7$

EXERCISE 4

a.	$\sqrt{100}$	=	10					
b.	$\sqrt{529}$	=	23					
c.	$\sqrt{231}$	=	15.2					
d.	$\sqrt{54.76}$	=	7.4					
e.	$\sqrt{702.25}$	=	26.5					
f.	$\sqrt{5643.21}$	=	75.12					
g.	$\sqrt{5+8.2}$	=	$\sqrt{13.2}$	=	3.63			
h.	$\sqrt{17.9-2.6+32.8}$	=	$\sqrt{48.1}$	=	6.94			
i.	$\sqrt{8}+\sqrt{15}$	=	2.83 + 3.87	=	6.7			
j.	$\sqrt{7^2+24^2}$	=	$\sqrt{49+576}$	=	$\sqrt{625}$	=	25	



NECA Pre-selection resources

Fractions - Parts of a whole

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FRACTIONS

A great number of trade calculations require an electrician to perform operations involving fractions. For example, the reading of scales, or the use of ratios when working with transformers.

LEARNING OUTCOME

- Can accurately perform operations that involve fractions

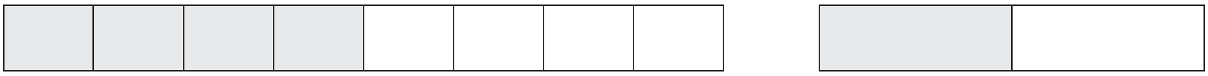
PERFORMANCE CRITERIA

- Expresses parts of a whole as a fraction.
- Reduces fractions to their simplest form.
- Uses the calculator to convert fractions into decimals.
- Uses the calculator to solve worded problems involving fractions.

FRACTIONS – PARTS OF A WHOLE

A great number of trade calculations require an electrician to perform operations involving fractions. For example, the reading of scales, or the use of ratios when working with transformers.

SIMPLIFYING FRACTIONS



$\frac{4}{8}$ **or** $\frac{1}{2}$

To reduce a fraction to its simplest form we look for a number that will divide evenly into the top (numerator) and the bottom (denominator) of the fraction.

To simply $\frac{4}{8}$ Divide 4 into the top and the bottom

$$\therefore \frac{4}{8} = \frac{1}{2}$$

The same fraction can appear in many different ways eg.

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{105}{210} = \frac{406}{812}$$

EXERCISE 1

Reduce these fractions to their simplest form:

a) $\frac{6}{9}$

b) $\frac{3}{12}$

c) $\frac{2}{10}$

d) $\frac{2}{14}$

e) $\frac{15}{25}$

f) $\frac{48}{72}$

☑ **Use the answer sheet to check your work.**

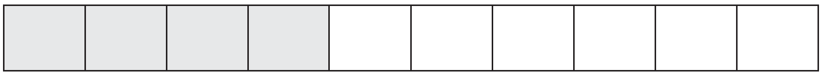
DECIMAL FRACTIONS

A fraction is a part of a whole e.g. 1/4

A decimal then is part of a whole and is often referred to as a DECIMAL FRACTION e.g. 1/4 = 0.25

A part or portion of a whole may be measured in either fractions or decimal fractions.

e.g.



The shaded area is $\frac{4}{10}$ but it is also 0.4 = 4 ÷ 10

Converting Fractions to Decimals

Express $\frac{3}{4}$ as a decimal fraction.

$\frac{3}{4}$ = 3 ÷ 4 = 0.75

Without a calculator:

Step 1. $4\overline{)3.00}$

Step 2. $\begin{array}{r} 0.00 \\ 4\overline{)3.00} \end{array}$ Divide 4 into 3. This won't go so, place a decimal point above the decimal.

Step 3. $\begin{array}{r} 0.70 \\ 4\overline{)3.00} \end{array}$ 4 goes into 30,7 times with 2 remainder.

Step 4. $\begin{array}{r} 0.75 \\ 4\overline{)3.00} \end{array}$ 4 goes into 20, 5 times.

EXERCISE 2

Convert the following fractions into decimals without using the calculator. Where possible, simplify the fractions before converting them.

a) $\frac{2}{5}$ = **b)** $\frac{9}{10}$ =

c) $\frac{6}{8}$ = **d)** $\frac{16}{20}$ =

e) $\frac{3}{8}$ =

Using the calculator

- **To convert a fraction to a decimal divide the dominator (bottom) into the numerator (top).**

Example	**eg.**	**7**	**÷**	**8**	**=**	**Answer 0.875**

- **Reciprocal Function (** $1/x$ **or** x^{-1} **)**

To take a reciprocal of a number means to divide that number into 1.

The reciprocal of 3 is 1/3

If your calculator has a reciprocal key ($^1/_x$ or x^{-1}) it can be used to convert fractions that have a numerator of 1 into decimals.

Depression of the reciprocal key causes the calculator to find the reciprocal of the number on the display. The reciprocal then appears on the display.

Example:

Express a $^1/_4$ as a decimal.

- $\boxed{4}$ $\boxed{^1/_x}$ **Answer : 0.25**

or

- $\boxed{4}$ $\boxed{x^{-1}}$ **Answer : 0.25**

The reciprocal of 4 is $^1/_4$

Steps on some calculators may differ. Refer to your calculator guide

EXERCISE 3

Convert the following fractions into decimals using the calculator (Give answers correct to 3 decimal places where appropriate).

a) $^{4}/_{7}$ =

b) $\frac{1}{12}$ =

c) $^{5}/_{8}$ =

d) $^{7}/_{11}$ =

e) $^{4}/_{26}$ =

f) $3\ ^{3}/_{4}$ =

g) $6\ ^{5}/_{8}$ =

h) $^{1}/_{5} + ^{1}/_{9}$

5	$^{1}/_{x}$	+	9	$^{1}/_{x}$	=	**Answer**

EXERCISE 4

It is helpful to memorize the decimal equivalents of the most used fractions.

Complete the table below by converting the fractions to decimals.

FRACTIONS	**DECIMALS** **(Correct to 2 dec. places)**
$\frac{1}{2}$	
$\frac{1}{3}$	
$\frac{2}{3}$	
$\frac{1}{4}$	
$\frac{3}{4}$	
$\frac{1}{5}$	
$\frac{1}{6}$	
$\frac{1}{8}$	
$\frac{1}{12}$	

EXERCISE 5

a) You need to order conduit to run along two walls. One wall measures 3 $\frac{1}{4}$ m and the other measures 6 $\frac{1}{2}$ m.

How much conduit do you need?

(Note: Conduit is plastic casing used to protect cables)

b) You have 15 x $\frac{1}{8}$ watt resistors.

What is the total power rating in watts of these resistors?

Express your answer in decimals.

c) How many lengths of cable 2 $\frac{1}{4}$ m can be cut from a piece of cable 18m long?

d) On a job an electrician uses $\frac{2}{3}$ of an 85m length of cable. How much cable is left?

☑ Use the answer sheet to check your work.

ANSWERS:

EXERCISE 1

a) $\dfrac{6}{9} = \dfrac{6 \div 3}{9 \div 3} = \dfrac{2}{3}$

b) $\dfrac{3}{12} = \dfrac{1}{4}$

c) $\dfrac{2}{10} = \dfrac{1}{5}$

d) $\dfrac{2}{14} = \dfrac{1}{7}$

e) $\dfrac{15}{25} = \dfrac{3}{5}$

f) $\dfrac{48}{72} = \dfrac{48 \div 12}{72 \div 12} = \dfrac{4}{6} = \dfrac{2}{3}$

EXERCISE 2

a) $\dfrac{2}{5} = 5\overset{\displaystyle 0.4}{\overline{\smash{)}2.0}} = 0.4$

b) $\dfrac{9}{10} = 0.9$

c) $\dfrac{6}{8} = \dfrac{6 \div 2}{8 \div 2} = \dfrac{3}{4} = 4\overset{\displaystyle 0.75}{\overline{\smash{)}3.00}} = 0.75$

d) $\dfrac{16}{20} = 0.8$

e) $\dfrac{3}{8} = 8\overset{\displaystyle 0.375}{\overline{\smash{)}3.000}} = 0.375$

EXERCISE 3

a)	$\frac{4}{7}$	=	0.571
b)	$\frac{1}{12}$	=	0.083
c)	$\frac{5}{8}$	=	0.625
d)	$\frac{7}{11}$	=	0.64
e)	$\frac{4}{26}$	=	0.154
f)	$3\,{}^{3}\!/_{4}$	=	3.75
g)	$6\frac{5}{8}$	=	6.625

h) ${}^{1}\!/_{5}$ + ${}^{1}\!/_{9}$ = 0.31 or 0.31

EXERCISE 4

Fractions	Decimals
$1/2$	0.5
$1/3$	0.33
$2/3$	0.67
$1/4$	0.25
$9/4$	0.75
$1/5$	0.2
$1/6$	0.17
$1/8$	0.13
$1/12$	0.08

EXERCISE 5

a) $3\,1/4 + 6\,1/2 = 3.25 + 6.5 = 9.75$

b) $15 \times 1/8 = 15 \times 0.125 =$ 1.875 watts

c) $18 + 2\,1/4 = 18 + 2.25 = 8$

Answer: 8 pieces of cable

d) $1/3 \times 85\text{m} =$ 28.33 metres or $28\,1/3$

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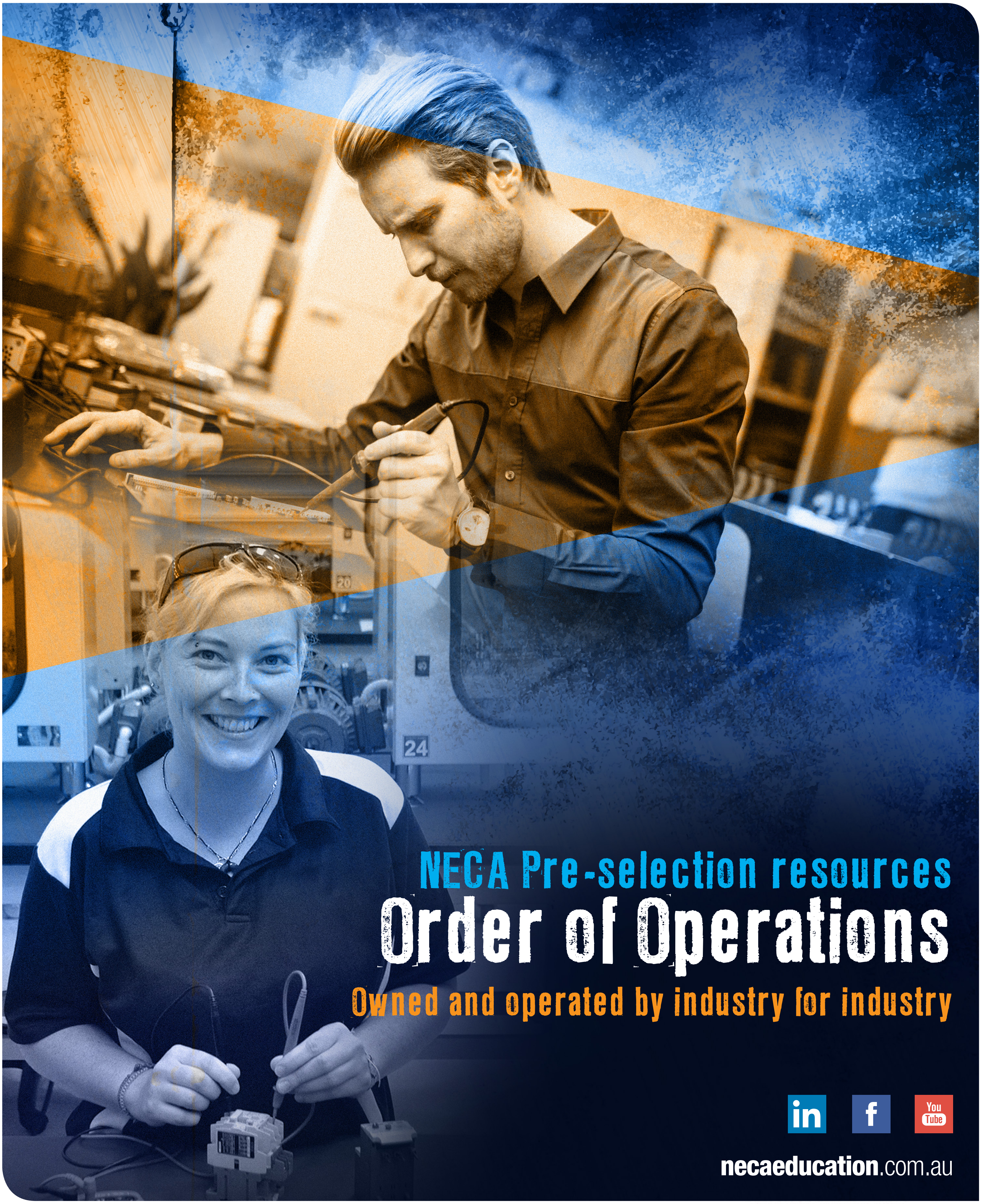

NECA Pre-selection resources

Order of Operations

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ORDER OF OPERATIONS

In mathematics the order in which we carry out the mathematical operations can have an effect on the result of the mathematical equation and we may end up with an incorrect result.

e.g.

6 - 4 x 3 = 6

(6-4) x 3

2 x 3 =6

Or

6 - 4 x 3 =-6

6 - (4 x 3)

6 - 12 = -6

A difference of 12 between the two results.

LEARNING OUTCOME

- Can understand and apply the mathematical rule for the order of operations.

PERFORMANCE CRITERIA

- Identifies the need to apply the order of operations
- Knows the rule for the order of operations
- Uses the rule to accurately work out the answer to problems

ORDER OF OPERATIONS

12 + 9 x 3 = ?

The correct answer is 39, not 63.

The order for working out the answer to problems of this type is determined by the rule

BODMAS or BOMDAS

That is:

Brackets
Division or **M**ultiplication

Addition or **S**ubtraction

The **BODMAS** rule tells you what you must do first:

1. Calculations inside the brackets

2. Then Divisions (÷) and Multiplication (x)

3. Then addition (+) and subtractions (-)

Note: If there are NO brackets, work the multiplications (x) and divisions (÷) first, then the additions (+) and subtractions (-).

Examples

1. 5 + 3 x 4

   First multiplication then addition

   = 5 + 12

   =17

2. 12 + 6 ÷ 2 - 5

   = 12 + 3 - 5

   First division, then addition and subtraction.

   =15 - 5

   =10

EXERCISE 1

Answer the following without using the calculator. Show your working.

a. $8 \times 8 + 4 =$

b. $6 \times 5 + 8 \times 3 =$

c. $27 \div 3 + 11 =$

d. $9 - 63 \div 9 + 14 \times 2 =$

e. $(9 - 6) \times (3 + 5) =$

f. $81 \div 9 \times (2 + 4) =$

☑ **Use the answer sheet to check your work.**

Using the calculator

Some calculators automatically take the BODMAS rule into account as you work from left to right.

Check your calculator by performing the following operation:

5 + 4 x 9 = 41

Note: If your calculator does not have bracket keys, you will need to perform the operations in brackets first, record your answers and then work through the calculations.

EXERCISE 2

Use your calculator to answer the following:

a) 6 + 4 ÷ 4 - 3 =

b) 62 + 64 ÷ 4 - 12 =

c) 8.2 x 8 + 4 =

d) (18 + 15 + 16) ÷ (63 ÷ 9) - 5 =

e) (25 x 7) x 3 ÷ (75 ÷ 15) =

☑ **Use the answer sheet to check your work.**

ANSWERS

EXERCISE 1

a) 8 x 8 + 4 = 64 + 4 = 68

b) 6 x5 + 8 x 3 = 30 + 24 = 54

c) 27 ÷ 3 + 11 = 9 + 11 = 20

d) 9 - 63 ÷ 9 + 14 x 2 = 9 - 7 + 28 = 30

e) (9 - 6) x (3 + 5) = 3 x 8 = 24

f) 81 ÷ 9 x (2 + 4) = 81 ÷ 9 x 6 = 54

EXERCISE 2

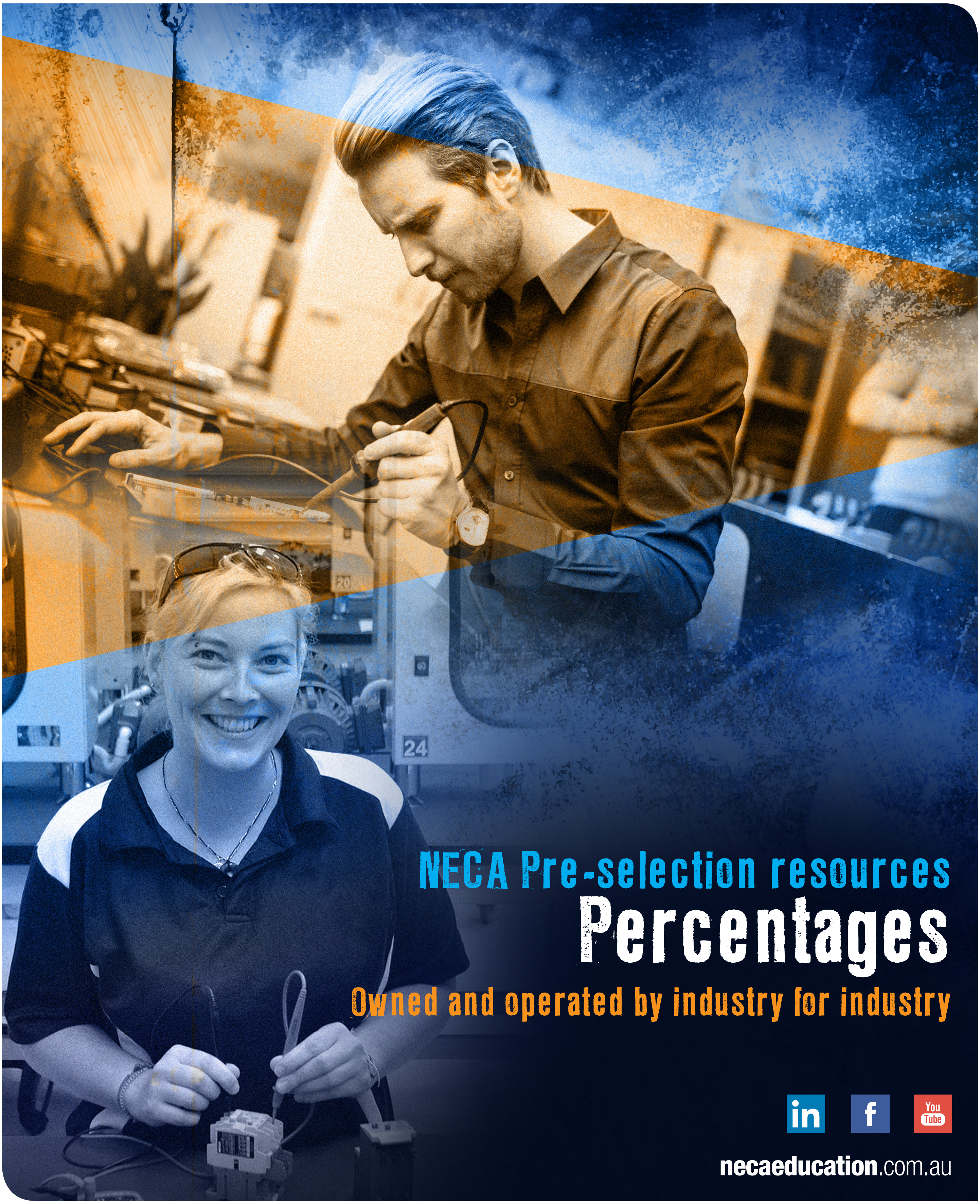
a) 6 + 4 ÷ 4 - 3 = 4

b) 62 + 64 ÷ 4 - 12 = 66

c) 8.2 x 8 + 4 = 69.6

d) (18 + 15 + 16) ÷ (63 ÷ 9) - 5 = 2

e) (25 x 7) x 3 ÷ (75 ÷ 15) = 105



NECA Pre-selection resources

Percentages

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PERCENTAGES

Percentage values are used extensively in the study of motors, generators and alternators. They are also applied in the study of resistors, transformers and time constraints.

For example, percentages are used to express the degree of power supply regulation, efficiency of a device, degree of slip in a motor motors and tapping of transformers.

LEARNING OUTCOME

- Can accurately calculate percentages

PERFORMANCE CRITERIA

- Expresses fractions and decimals as a percentage.
- Uses the calculator to accurately express one quantity as a percentage of another.
- Uses the calculator to accurately calculate the percentage of a value.
- Uses the calculator to accurately calculate percentage increase and decrease.
- Uses the calculator to solve electrical problems involving percentage calculations.

PRELIMINARY EXERCISE

Multiplying and Dividing by 10's

When multiplying by multiples of ten, the decimal point is moved over the same number of places as there are zeros in the multiplier, eg.:

3.54 x 10 = 35.4	(One zero implies one place).
3.54 x 100 = 354	(Two zeros implies two places).
3.54 x 1000 = 3540	(Three zeros implies three places).

EXERCISE 1

Calculate answers to the following without using a calculator.

a)	0.73 x 10 =	b)	0.73 x 1000 =
c)	12.60 x 100 =	d)	0.0089 x 10 =
e)	761.2 x 100 =	f)	3.504 x 100 =
g)	.6821 x 10000 =		

When dividing by multiples of ten, the decimal point moves to the left. Again, the same number of places as there are zeros in the divisor,
eg.

2.67 ÷ 10 = 0.267	398.6 ÷ 10 = 39.86
2.67 ÷ 100 = 0.0267	398.6 ÷ 100 = 3.986
2.67 ÷ 1000 = 0.00267	398.6 ÷ 1000 = .3986

EXERCISE 2

Using the calculator

Calculate answers to the following using the calculator.

a)	0.087 ÷ 10 =	b)	56.8 ÷ 100 =
c)	156.7 ÷ 1000 =	d)	2.4 ÷ 100 =
e)	0.003 ÷ 10 =	f)	100.67 ÷ 100 =

☑ **Use the answer sheet to check your work.**

WHAT IS A PERCENTAGE?

A percentage is a special fraction. Per cent means out of one hundred or per 100.

Percentages are fractions with 100 on the bottom (the denominator).

Hence: $7\% = \frac{7}{100}$

$50\% = \frac{50}{100} = \frac{1}{2}$

$100 = \frac{100}{100} = 1$ ('everything')

EXERCISE 3

Match each of the short statements below with the most likely percentage. Draw a line to link the statement and the percentage.

Blood Alcohol Content	9.2%
Time and Half Overtime Rate	3%
Totally Fat Free	.05%
All Steel Construction	150%
Tax Rate	0%
A Wage Rise	100%
Operating on Half Load	50%
Superannuation Contribution	33%

PERCENTAGE DIAGRAMS

EXERCISE 4

Complete the following statements:

a) 8 per cent means $\frac{\_\_\_\_\_\_}{100}$ or .... parts in 100 parts

b) 4 per cent means .... in 100

c) 3.9 per cent means $\frac{\_\_\_\_\_\_}{100}$

d) 5% = $\frac{\_\_\_\_\_\_}{100}$ = $\frac{\_\_\_\_\_\_}{20}$ (cancelling by 5's)

EXERCISE 5

Write the following percentages as simple fractions (cancel down the answer where possible).

a) 25% =

b) 60% =

c) 11% =

d) $12^{1/}{}_{2}\% = \frac{25\%}{2}$

$= \frac{25}{2} \times \frac{1}{100}$

$= \frac{25}{2x100}$

=

=

e) $33^{1/}{}_{3}\% = \frac{...\%}{3}$

$= \frac{}{3 \times 100}$

f) $7^{1}/_{4}\% =$

☑ **Use the answer sheet to check your work.**

WRITING FRACTIONS AND DECIMALS AS PERCENTAGES

To express a fraction or decimal as a percentage simply multiply by 100 and put the % sign in place.

Example 1 - Decimals to Percentages

a) 0.31 as a percentage 0.31 x 100% = 31%

Using the calculator

0	.	3	1	x	1	0	0	=	Answer 31%

b) 0.8 as a percentage 0.8 = 0.8 x 1000k = 80%

Using the calculator

0	.	8	x	1	0	0	=	Answer 80%

Example 2 - Fractions to Percentages

a) Write 1/4 as a percentage:

1/4 = $\frac{1}{4} \times \frac{100}{1}$

= $\frac{100}{4}$

\- = 25%

Using the calculator

1	÷	4	x	1	0	0	=	Answer 25%

b) Write $\frac{45}{50}$ as a percentage:

$\frac{45}{50}$ = $\frac{45}{\cancel{50}} \times \frac{\overset{2}{\cancel{100}}}{1}\ \%$

= $\frac{45}{1} \times \frac{2}{1}\ \%$

= 90%

Using the calculator

4	5	÷	5	0	x	1	0	0	=	Answer 90

or

Using the calculator

4	5	÷	5	0	%	=	Answer 90

Note: Steps used on some calculators may differ. Refer to your calculator guide.

EXERCISE 6

Write these fractions as percentages using the calculator. (Correct to 2 decimal places where appropriate)

a) $\frac{1}{10}$ =

b) $\frac{5}{8}$ =

c) $\frac{7}{9}$ =

d) $\frac{45}{80}$ =

EXERCISE 7

Using the calculator, complete the following table of commonly used fractions and percentages and try to remember them.

FRACTION	PERCENTAGE
1/4	
1/2	
3/4	
1/5	
2/5	
3/5	
4/5	
1/8	
3/8	
1/3	
2/3	
1/6	
1/10	
1/20	

☑ **Use the answer sheet to check your work.**

EXPRESSING ONE QUANTITY AS A PERCENTAGE OF ANOTHER

If we want to express the ratio of one quantity to another as a percentage then we must first record the two quantities as a fraction.

Example 3

Of 760 light fittings produced, 80 had defects. What percentage had defects?

$$\frac{80}{760}\times 100$$

$$= \frac{8000}{760}$$

= 10.53% (two decimal places)

Note: Steps used on some calculators may differ. Refer to your calculator guide.

8	0	÷	7	6	0	%	=	Answer 10.53%

EXERCISE 8

Express the following as percentages:

a) 26 out of 62

b) 13 out of 27

c) 76 out of 220

d) 3 out of 75

e) 40 is what percentage of 320?

f) 184 as a percentage of 73

☑ **Use the answer sheet to check your work.**

EFFICIENCY

Efficiency of an electrical device can be calculated using the following formula:

$$\text{Efficiency} = \frac{\text{Power Output} \times 100}{\text{Power Input}}$$

$$= \frac{\text{Power Output } \%}{\text{Power Input}}$$

The power output is expressed as a percentage of the power input.

This tells us how efficiently the device is using the available power. The higher the percentage, the more efficient the device.

Efficiency Formula

Efficiency

$= P_{out} / P_{in} \times 100\%$

$= P_{out} / P_{in}\ \%$

Example 4

If a mechanical device has a power input of 160W and a power output of 120W, find the efficiency.

$$\text{Efficiency} = \frac{120}{160} \times 100 = 75\%$$

Using the calculator

1 2 0 ÷ 1 6 0 % Answer 75%

EXERCISE 9

Find the efficiency of the following devices given the power output and input.

a) Amplifier
Output = 100W Input = 250W

b) Electric Motor
Output = 3kW Input = 3.35kW

c) Radiator
Output = 1000W Input = 1010W

d) Solar Cell
Output = 1W Input = 5W

EXERCISE 10

List the devices in Exercise 10 in order of efficiency from most efficient to least efficient.

1. …….
2. …….
3. …….
4. …….

☑ **Use the answer sheet to check your work.**

FINDING A PERCENTAGE OF A VALUE

Example 5

12% of $250

= $\frac{12}{100} x \frac{250}{1}$

= $30

Using the calculator

2	5	0	x	1	2	%	=	Answer 30

-

or

1	2	%	x	2	5	0	=	Answer 30

EXERCISE 11

Calculate the following:

a) 45% of 300 =

b) 140% of 0.05 =

c) 65% of 1200 =

d) 0.020% of 1500kHz =

EXERCISE 12

If a company must reduce its workforce of 670 by 20%, how many workers must be retrenched?

EXERCISE 13

If you earn 5% commission on sales of $2,000, how much commission do you earn?

☑ **Use the answer sheet to check your work.**

PERCENTAGE INCREASE AND DECREASE

Quantities such as changes in amperage, increases in voltage and ranges in resistance are often expressed as percentage changes.

A cable is overloaded by 26%.
The amperage of a current in a circuit decreases by 25%.
A resistor is labelled as having a resistance of 22Ohms plus or minus 5%.

Example 6
A multimeter is priced at $250 plus 20% sales tax. How much do you have to pay for the multimeter?

The increase is given as a percentage of the original, so the increase in this example is 20% of $250:

$$\frac{20}{100} \times 250$$

$$= \frac{2\cancel{0}}{10\cancel{0}\cancel{0}} \times 25\cancel{0}$$

= 2 x 25

= $50

Using the calculator
Note: Steps used on some calculators may differ. Refer to your calculator guide.

2	5	0	x	2	0	%	=	Answer $50

$\therefore$ The new expenditure = original expenditure + increase

= $250 + $50
= $300

Example 7
At a switchboard the voltage is 240V. At the end of a circuit which is fed by the switchboard, the voltage has dropped by 5%.
What is the voltage at the end of the circuit?

5% of 240V

$$\frac{5}{100} x240 = \frac{5}{10\cancel{0}} x24\cancel{0}$$

=12V

$\therefore$ Voltage at the end of the circuit
= 240V-12V
= 228V

EXERCISE 14

The power output of a broadcast station is increased by 40%. If the original power output was 1000W what is the new output?

EXERCISE 15

A resistor is marked as being 47,000 ohms ± 10%.

What is the maximum acceptable value of the resistor?

What is the minimum acceptable value of the resistor?

EXERCISE 16

A resistor is labelled as having a resistance of 22 ohms plus or minus 5%. When measured the same resistor is found to have an actual resistance of 21 ohms.

Is this value acceptable? (Show your working)

☑ **Use the answer sheet to check your work.**

PERCENTAGE CHANGE

A useful way to examine the change in size of quantity is to calculate its increase or decrease as a percentage of its original size.

In some instances it is necessary to find the percentage of change of some electronic characteristic.

This can be calculated by using the following formula.

$$\% \text{ of change} = \frac{change \times 100}{original\,value}$$

Example 8

The voltage is increased from 100 to 125 volts.
What is the percentage of increase?

$$\% \text{ of change} = \frac{125-100}{100} \times 100$$

$$= \frac{25}{100} \times 100$$

$$= 25\%$$

Using the calculator

Note: Steps used on some calculators may differ. Refer to your calculator guide.

$\boxed{1}$ $\boxed{2}$ $\boxed{5}$ $\boxed{-}$ $\boxed{1}$ $\boxed{0}$ $\boxed{0}$ $\boxed{=}$ Answer $25

Then

$\boxed{\div}$ $\boxed{1}$ $\boxed{0}$ $\boxed{0}$ $\boxed{\%}$ Answer 25%

Answer The voltage has increased by 25%.

Example 9

If the voltage is decreased from 125 to 100.
What is the percentage of decrease?

% of change = $\frac{125-100}{125}\times 100$

= $\frac{25}{125}\times 100$

= 20%

Using the calculator

1	2	5	-	1	0	0	=	Answer 25

Then

÷	1	2	5	%	Answer 20%

Answer The voltage has decreased by 20%

EXERCISE 17

Current in a circuit decreases from 8 to 6 amperes.
What is the percentage of decrease?

EXERCISE 18

A cable is rated to carry up to 50 amps. It is however measured to be carrying 63 amps. By what percentage is the cable overloaded?

EXERCISE 19

A transformer is used to step up voltage from 240 volts to 420 volts. By what percentage has the voltage been increased?

EXERCISE 20

Voltage in a Circuit is increased from 120 to 130 volts. What is the percentage of increase?

PERCENTAGE OF ERROR

In some situations an electrician needs to know percentage of error. For example, suppose that the calculated value of a quantity is 60 volts but the measured value is 66 volts.

% error = $\frac{difference}{reference\ value} \times 100$

= $\frac{66-60}{60} \times 100$

= $\frac{\cancel{6}}{\cancel{6}\cancel{0}} \times 10\cancel{0}$

= 10%

Answer
The percentage error in the measured voltage value is 10% (too high.)

EXERCISE 21

Resistance in a circuit should be 50,000 ohms, but the actual value is 48,000 ohms.
What is the percentage of error?

EXERCISE 22

Voltage in a circuit is 120 volts, but it should be 128 Volts. What is the percentage of error?

☑ **Use the answer sheet to check your work.**

PERCENTAGE RULES

1. Percentage is a method of writing hundredths as whole numbers.

Example $\frac{63}{100}$ is 63%; In reverse, 63% is $\frac{63}{100}$ or .63

a) $\frac{5}{100}$ = 5% = .05

b) 19% = $\frac{19}{100}$ = .19

c) .89 = $\frac{89}{100}$ = 89%

100

The whole of anything Is 100% or $\frac{100}{100}$ or 1

2. Decimals are changed to percent by multiplying by 100 and adding the "%" sign. This is the same as moving the decimal point two places to the right.

a) .77 =.77 x 100% = 77%

b) 1.05 = 1.05 x 100% =105%

c) .002 = .002 x 100% = .2%

3. Fractions are changed to percent by first changing to a decimal then use the procedure outlined In (2).

$\frac{7}{8}$ = 7 ÷ 8= .875 or 87.5%

4. Percent is changed to a decimal by dividing by 100. This is the same as moving the decimal point two places to the left and dropping the "%" sign.

a) 93% = .93

b) 140% = 1.40

c) .7% = .007

5. To find what percent one number is of another: Establish a fraction - the part is the numerator; the whole is the denominator. Express this fraction as a percentage.

Example

An apprentice was asked to Install 80 metres of cable on a building site. At the end of the day the apprentice had Installed 60 metres.
What percent of the cable did he Install?

$\frac{60}{80}$ = 60 $\div$ 80 = .75 = 75%

6. To find what a certain percent of a number Is, change the percent to a decimal and multiply.

Example

What is 20% of 600?

20% x 600 = $\frac{20}{100}$ x 600 =.20 x 600 = 120

ANSWERS

EXERCISE 1

a) 0.73 x 10 = 7.3

b) 0.73 x 1000 = 730

c) 12.60 x 100 = 1260

d) .0089 x 10 = 0.089

e) 761.2 x 100 = 76120

f) 3.504 x 100 = 350.4

g) .6821 x 10000 = 6821

EXERCISE 2

a) .087 + 10 = 0.0087

b) 56.8 + 100 = 0.568

c) 156.7 + 1000 = 0.1567

d) 2.4 + 100 = 0.024

e) 0.003 + 10 = 0.0003

f) 100.67 + 100 = 1.0067

EXERCISE 3

Blood Alcohol Content = .05%

Time and Half Overtime Rate = 150%

Totally Fat Free = 0%

All Steel Construction = 100%

Tax Rate = 33%

A Wage Rise = 3%

Operating on Half Load = 50%

Superannuation Contribution = 9.2%

EXERCISE 4

a) 8% means $\frac{8}{100}$ or 8 parts in 100

b) 4% means 4 parts in 100

c) 3.9% means $\frac{3.9}{100}$

d) 5% means $\frac{5}{100} = \frac{1}{20}$

EXERCISE 5

a) 25% = $\frac{25}{100}$ = $\frac{1}{4}$

b) 60%= $\frac{60}{100} = \frac{3}{5}$

c) 11% = $\frac{11}{100}$

100

d) 12 $\frac{1}{2}$ % = $\frac{25}{200} = \frac{1}{8}$

e) 33 $\frac{1}{3}$ %= $\frac{100\%}{3} = \frac{100}{3x100} = \frac{100}{300} = \frac{1}{3}$

f) 7 $\frac{1}{4}$ % = $\frac{29\%}{4} = \frac{29}{4x100} = \frac{29}{400}$

EXERCISE 6

a) $\frac{1}{10}$ = 10%

b) $\frac{5}{8}$ = 62.5%

c) $\frac{7}{9}$ = 77.78%

d) $\frac{45}{80}$ = 56.25%

EXERCISE 7

1/4 = 25%

1/2 = 50%

3/4 = 75%

1/5 = 20%

2/5 =40%

3/5 =60%

4/5 =80%

1/8 = 12.5%

3/8 = 37.5%

1/3 = 33.33%

2/3 = 66.67%

1/6 = 16.67%

1/10 = 10%

1/20 = 5%

EXERCISE 8

a) $\frac{26}{62}$ x 100 = 41.94%

b) $\frac{13}{27}$ x 100 = 48.15%

c) $\frac{76}{220}$ x 100 = 34.55%

d) $\frac{3}{75}$ x 100 = 4%

e) $\frac{40}{320}$ x 100 = 12.5%

f) $\frac{184}{73}$ x 100 = 252.05%

EXERCISE 9

a) Efficiency = $\frac{100x100}{250}$ = 40%

b) $\frac{3x100}{3.35}$ = 89.55%

c) $\frac{1000x1000}{1010}$ = 99%

d) $\frac{1x100}{5}$ = 20%

EXERCISE 10

1) Radiator

2) Electric Motor

3) Amplifier

4) Solar Cell

EXERCISE 11

a) 45% of 300 = 135

b) 14% of 0.05 = 0.007

c) 65% of 1200 = 780

d) 0.02% of 1500kHz = 0.3kHz

EXERCISE 12

20% of 670 = 134 workers

EXERCISE 13

5% of $2000 = $100

EXERCISE 14

40% of 1000

$= \frac{40}{100} x \frac{1000}{1} = 400W$

New Output = 1000W ÷ 400W

= 1400W

EXERCISE 15

10% of 47000

$= \frac{10}{100} x \frac{47000}{1} = 4700$

Maximum acceptable value of resistor = 47000 ÷ 4700

= 51700ohms

Minimum acceptable value of resistor

= 47000 - 4700

= 42300ohms

EXERCISE 16

5% of 22

= $\frac{5}{100} x \frac{22}{1} = 1.1$

Therefore the acceptable range of resistance
= 22 ÷ 1.1 to 22 - 1.1
= 23.1 to 20.9

A resistance of 21ohms lies within this range, therefore this is an acceptable value.

EXERCISE 17

% of change

= $\frac{8-6}{8} x1\ 0$

= $\frac{2}{8} x100 = 25\%$

The current has decreased by 25%

EXERCISE 18

% of change

= $\frac{63-50}{50} x100$

= $\frac{13}{50} x100 = 26$

The cable is overloaded by 26%

EXERCISE 19

% of change

= $\frac{420-240}{240} x100$

= $\frac{180}{240} x100 = 75$

The voltage has been increased by 75%

EXERCISE 20

% of change

$$= \frac{130-120}{120} x100$$

$$= \frac{10}{120} x100 = 8.33$$

The voltage has increased by 8.33%

EXERCISE 21

% error

$$= \frac{50000-48000}{50000} x100$$

$$= \frac{200}{50000} x100$$

= 4

The percentage error in the resistance is 4% too low

EXERCISE 22

% error

$$= \frac{128-120}{128} x100$$

$$= \frac{8}{128} x100 = 6.25$$

The percentage error in measured voltage is 6.25% too low

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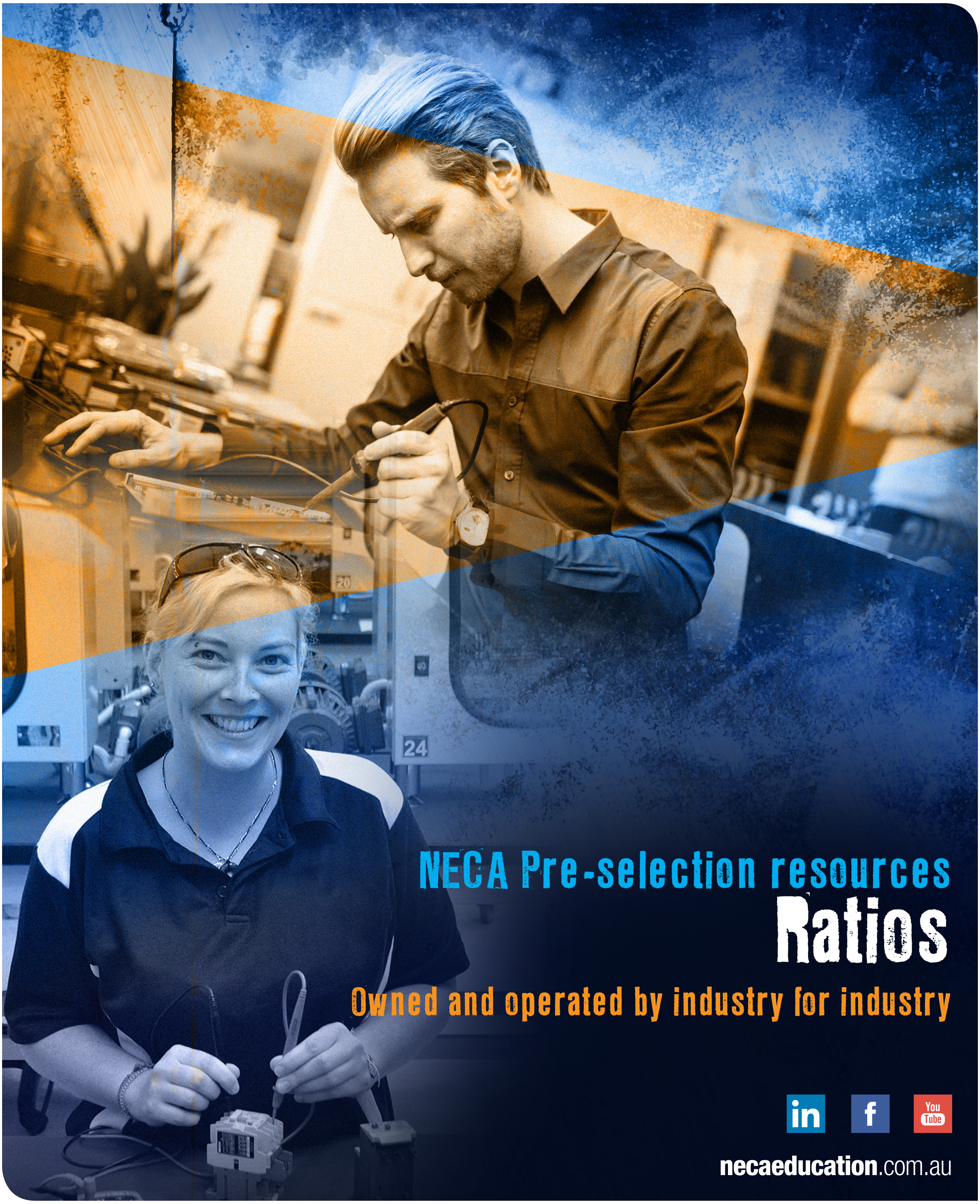

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Ratios

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RATIOS

An understanding of ratios is required for many areas of electrical theory.

Examples of the application of ratios in the electrical trade include:

- calculating the value of current flow in a circuit. This is dependent upon the ratio between applied voltage and circuit impedance
- determining power factor. This is calculated by forming a ration between true power and apparent power.
- calculating the output voltage of a transformer given the ration between the input and output voltages.

LEARNING OUTCOME

- Can make comparisons using the mathematical expression of ratio.

PERFORMANCE CRITERIA

- Uses a stated order to express ratio.
- Expresses ratio in its simplest form.
- Uses ratio to determine a specific quantity by working backwards.
- Understands the relationship between common and decimal fractions and percentages.

WHAT IS A RATIO?

A ratio is a mathematical way of comparing quantities. This comparison is made in a stated order.

Example 1

In an electrical company there are 30 employees. 20 are male and 10 are female. Form a ratio of male to female staff by completing this ratio.

20 : 10

Number of Males

Number of Females

Statements that can be used to express this ratio are:

- There are twice as many males as females
- There are half as many females as males
- For every 20 males there are 10 females

EXPRESSING A RATIO IN ITS SIMPLEST FORM

Example 2

Two light globes are rated at 80W and 40W. The ratings are in the ratio 80:40. However, it is usual to express ratios, like fractions, in their lowest terms.

Therefore	80 : 40	
is the same as	8 : 4	(dividing both sides by 10)
and	2 : 1	(dividing both sides by 4)

The ratio of the power ratings of the globes, when expressed in its simplest form becomes 2: 1.

Example 3

If one motor rotates at 300rpm (revolutions per minute) and the other at 750rpm, what is the ratio of their speeds?

	300 : 750	
=	6 : 15	(cancelling by 50)
=	2 : 5	(cancelling by 3)

Sometimes it may be clearer to have one side of the ratio to 1.

2 : 5	
1 : 2.5	(dividing both sides by 2)

Therefore, for every one revolution of the first machine the second machine makes 2.5 revolutions.

EXERCISE 1

Express the following ratios as simply as possible:

a) 25A to 100A (A = amperes)

b) 100V to 12.5V (V = volts)

c) 300 mins to 6 mins

EXERCISE 2

A torch globe draws 150mA. A transistor radio draws 600mA. Express the ratio of the radio current to the globe current.

EXERCISE 3

A transformer has an input voltage of 240 volts and an output voltage of 12 volts.

a) Express the transformer ratio in its simplest form.

b) Complete the statement

For every ……………volts applied at the input to the transformer

one volt is produced at the output.

EXERCISE 4

On a clear day the earth receives 1200 watts of power for every square metre of surface area. A bank of one square metre of cells generates 120 watts.

a) What is the ratio of sun power to solar cell power?

b) Complete the statement:

For every 1 watt of solar cell power we need……………………..watts of

sun power.

☑ **Use the answer sheet to check your work.**

METRIC EQUIVALENTS

If you are forming a ratio between units of the same type ego length, power, weight or current, then convert them into the same form.

The table below gives the metric equivalents commonly used in the electrical trade.

W = watts	M = mega
V = volts	k = kilo
A = amperes	m = milli
Ω = ohms	

Metric Equivalents

1 MW=1000kW	1 MA = 1000 kA
1 kW =1000W	1 kA= 1000 A
1 W =1000mW	1 A = 1000 mA
1 MV = 1000 kV	1 MΩ = 1000 kΩ
1 kV = 1000 V	1 kΩ = 1000 Ω
1 V = 1000 mV	1 Ω = 1000 rnΩ

Example 4

What is the ratio between 20 Volts (V) and 5 kilovolts (kV)?

Before cancelling down the ratio it is necessary to express both parts of the ratio in the same terms by converting the kilovolts to volts.

	20 V : 5 kV	
=	20 V : 5000 V	(changing kV to V)
=	1 : 250	(dividing both sides by 20)

EXERCISE 5

Express the following ratios as simply as possible using only whole numbers:

a) 800 W to 550 kW

b) 45 seconds to 1 minute 30 seconds

c) 24 kW to 84 MW

d) 24 mΩ to 25 Ω

e) 5 mV to 2.5 V.

EXERCISE 6

The rated power input of a device is the maximum power at which the device is designed to operate.

The headphone of the walkman has a rated power input of 120 mW.

The rated power input of a small stereo speaker is 8 W.

a) What is the ratio of the rated power inputs of the speakers to the headphones?

b) Complete the statement:

In the above example …………………….headphones would consume the same amount of power as one stereo speaker?

EXERCISE 7

A multimeter is an instrument used to measure voltage, current and resistance. To find a fault in a circuit a multimeter is used to check the resistances. The multimeter measured 6.8 kΩ at one place in the circuit and 400 Ω at another place in the same circuit.

What is the ratio between the two resistances?

USING RATIOS – WORKING BACKWARDS

If a ratio is known then calculations can be made to determine a specific quantity. The most common application of ratios in electrical theory is in calculating input and output voltages of transformers.

- The input voltage is also called the primary voltage
- The output voltage is also called the secondary voltage

Example 5

A step-down transformer has an input/output ratio of 2 : 1. If the output voltage is 60 volts, what is the input voltage?

	2	:	1
	Input		Output
=	120V	:	60V
	(2 x 60) :		(1 x 60)
	input		output

Answer: The input or primary voltage of the step-down transformer is 120 Volts.

Example 6

A step-down transformer has an input/output ratio of 20: 1. If the input voltage is 240 volts, what is the output voltage?

	20	:	1
	Input		Output
=	240V	:	12V
			(240÷20)
	Input		Output

Answer: The output or secondary voltage of the transformer is 12 volts.

EXERCISE 8

A **step-down** transformer has a ratio of 6 : 1. If the output voltage is 40V, what is the input voltage?

Answer: The input voltage of the transformer is……………………volts.

EXERCISE 9

A **step-down** transformer is operated from a 120 volt line. If the voltage ratio of the transformer is 10 : 1, what is the secondary voltage.

Answer: The secondary voltage of the transformer is……………………volts.

EXERCISE 10

A **step-up** power transformer has a ratio of 1 : 7 and is operated from a 115 volt line. What is the secondary voltage?

Answer: The secondary voltage of the transformer is ....…………….volts.

EXERCISE 11

A **step-up** power transformer has an output voltage of 480V. If the ratio of the transformer is 1 : 12, what is the voltage of the line that operates the transformer?

☑ **Use the answer sheet to check your work.**

RATIOS / FRACTIONS / PERCENTAGES

Ratios can be expressed as common and decimal fractions and as percentages.

Decimal fractions are special fractions whose denominators are always 10 or multiples of ten.

Percentages are special fractions whose denominators are always 100.

RATIOS: EQUIVALENT FRACTIONS AND PERCENTAGES

RATIO	COMMON FRACTION	DECIMAL FRACTION		PERCENTAGE	
1:2	1/2	0.5	$\frac{5}{10}$	50%	$\frac{5}{100}$
1:4	1/4	0.25	$\frac{25}{100}$	25%	$\frac{25}{100}$
1:8	1/8	0.125	$\frac{125}{1000}$	12.5%	$\frac{12.5}{100}$
3:4	3/4	0.75	$\frac{75}{100}$	75%	$\frac{75}{100}$
1:3	1/3	0.33	$\frac{33.3}{100}$	33.3%	$\frac{33.3}{100}$
2:3	2/3	0.66	$\frac{66.6}{100}$	66.6%	$\frac{66.6}{100}$
1:10	1/10	0.1	$\frac{1}{10}$	10%	$\frac{10}{100}$
1:20	1/20	0.05	$\frac{5}{100}$	5%	$\frac{5}{100}$
1:5	1/5	0.2	$\frac{2}{10}$	20%	$\frac{20}{100}$
2:5	2/5	0.4	$\frac{4}{10}$	40%	$\frac{40}{100}$
3:10	3/10	0.3	$\frac{3}{10}$	30%	$\frac{30}{100}$

ANSWERS:

EXERCISE 1

a) 25:100 = 1:4

b) 100:12.5 (dividing both sides by 12.5)

= 8:1

c) 300:6

= 50:1

EXERCISE 2

radio current: globe current

= 600:150

= 4:1 (dividing both sides by 150)

EXERCISE 3

a) 240:12 (dividing both sides by 12)

Input output

= 20:1

b) For every **20** volts applied at the input to the transformer, 1 volt is produced at the output.

EXERCISE 4

a) sun power: solar cell power

= 1200:120 (dividing both sides by 120)

= 10:1

b) For every 1 watt of solar cell power we need **10** watts of sun power.

EXERCISE 5

a) 800W:550kW

= 800W:550000W (changing kW to W)

= 8:5500 (dividing both sides by 100)

= 2: 1375 (dividing both sides by 4)

b) 45 secs:1 min 30 secs

= 45 secs:90 secs (changing mins to secs)

= 1:2 (dividing both sides by 45)

c) 24kW: 84 MW

24:84000 (changing MW to kW)

1:3500 (dividing both sides by 24)

d) 24mΩ:25Ω

24mΩ:25000mΩ (changing Ω to mΩ)

3: 3125 (dividing both sides by 8)

e) 5mV: 2.5V

5mV: 2500mV (changing V to mV)

1:500 (dividing both sides by 5)

EXERCISE 6

a) 8W: 120mW

= 8000mW: 120mW

= 200:3

b) 66 $\frac{2}{3}$ headphones would consume the same amount of power as one stereo speaker

EXERCISE 7

6.8kΩ:400Ω

= 6.8 x 1000Ω:400Ω

= 6800:400

= 17:1

EXERCISE 8

The input voltage of the transformer is **240** volts

6:1

Input: Output

240V:40V

EXERCISE 9

The secondary voltage of the transformer is **12** volts

10: 1

Input : Output

120V: 12V

EXERCISE 10

The secondary voltage

1:7

Input: Output

115V: 805V

EXERCISE 11

1:12

Input Output

40V : 480V

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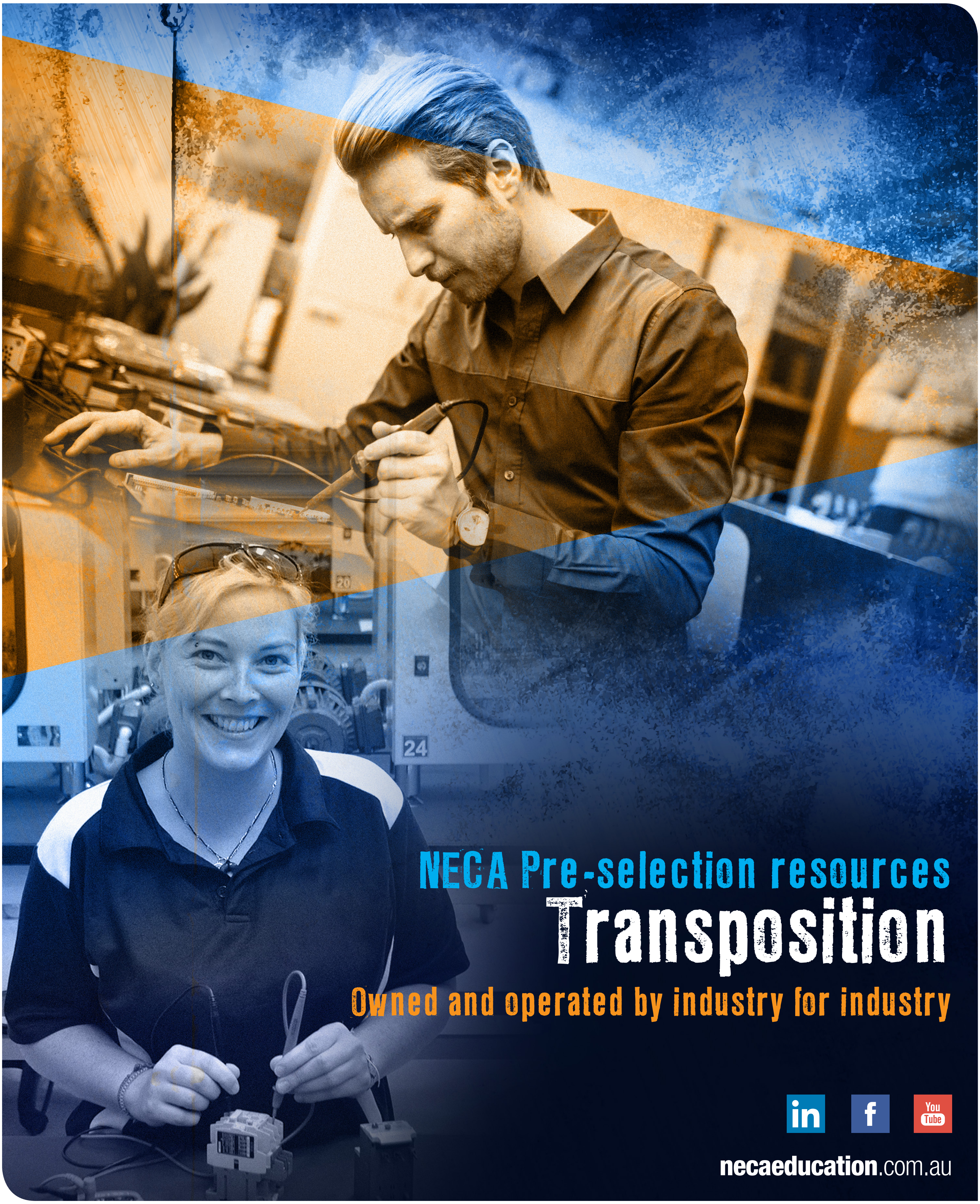

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TRANSPOSITION

A large portion of practical mathematics in Electrical studies consists of working with electrical formulas (equations)

These formulae are algebraic expressions showing how some value varies with respect to others.

Often the formula is given in such a way that to use the available data you need to rearrange or transpose the formula. Electrical formulae are used to calculate values such as : circuit impedance, resistance, capacitance, applied voltages, efficiency of transformers and power ratings.

LEARNING OUTCOME

- Can transpose formulae to solve for an unknown value.

PERFORMANCE CRITERIA

- Simplifies and expands algebraic expressions.
- Substitutes values in algebraic expressions to find a solution.
- Understands the rules of transposition.
- Uses transposition to solve equations.
- Transposes electrical formulae to find an unknown value.

PART A

REVIEW OF BASIC ALGEBRA

In order to use and transpose formulae it is necessary to understand how to perform operations using algebra.

Algebra is essentially the mathematics of symbols with the laws of arithmetic being applied to letters instead of numbers. The letters or symbols can stand for a number.

For Example: 7b 2zy 1R

Note: 7b means 7 x b

2zy means 2 x z x y

1R means 1 x R

In algebra the **multiplication** sign is not written.

Grouping Like Terms

When **adding** and **subtracting** in algebra only the terms containing the same symbols can be added or subtracted.

Example 1

2a + 4b + 3a
= (2a + 3a) + 4b
= 5a + 4b

The 'a' terms have been added together

Example 2

7r + 3c

This cannot be simplified because the pronumerals (represented by the letters 'r' and 'c') are different.

It is like trying to add 7 resistors and 3 capacitors. The combined result is still 7 resistors and 3 capacitors.

Example 3

9y-3y = 6y

Example 4

6w - 4z + 10

This cannot be simplified.

EXERCISE 1

Simplify the following where possible:

a) $4A+3A$

b) $3x-y$

c) $8m - 7m$

d) $2p + 9a + 4p - 2a$

e) $6R1 - 4R2 + 3R1$

f) $26w - 26w$

g) $4V + 6W - 2V + 8Z$

☑ **Use the answer sheet to check your work.**

Example 5 6xy + 5xw

This cannot be simplified

Example 6 2efg + 3efg + 4fhg = 5efg + 4fhg

Example 7 9xy - 4yx

= 9xy - 4xy

= 5xy

Note: xy is the same as yx. The order of the letters does not matter.

EXERCISE 2

a) 4ab+ 6ab

b) xy + 5xy

c) 8pqr - 7pqr

d) 5ef + 2fg - 3ef

e) 15mn -15mn

f) 8wx + 3xw

g) $9R_1\,R_2 + 7L_1\,L_2 - 5R_1\,R_2 + 3L_2\,L_1$

h) 2ghi + 3igh + 9hig - 2hji

☑ **Use the answer sheet to check your work.**

Example 8 $f \times g \times h = fgh$

Example 9 $3z \times 4y = 12zy$

EXERCISE 3

Rewrite the following. You will need to remove the multiplication sign.

a) $p \times q \times r$

b) $3m \times n$

c) $6h \times 5l$

d) $2R1 \times 4R2$

e) $5w \times 2z \times 3y$

Expanding the brackets and grouping like terms:

Example 10 $2(x - y) = 2x - 2y$

Example 11 $7(a - 2b) = 7 \times a - 7 \times 2b$

$= 7a - 14b$

Example 12 $4(2F - G) + 6F = 8F - 4G + 6F$

$= 14F - 4G$

Example 13 $p(3m - 3r) - m(2p + 4r)$

$= 3pm - 3pr - 2mp - 4mr$

$= 3pm - 2mp - 3pr - 4mr$

$= pm - 3pr - 4mr$

EXERCISE 4

Simplify the following:

a) $3(a + b)$

b) $5(m - n)$

c) $12(p - 2q)$

d) $4(2R1 + 3R2) - 3R2$

e) $3(r + s) + 4(2r - s)$

f) $6(m + 2n) + 2(4m + n)$

g) $8(3s + 2t) + 6(2s - 3t)$

h) $4(3x - y) - (x - y)$

(Remember: - x- = +, - x+ =-)

I) $3r(s + 2q) + rq$

☑ **Use the answer sheet to check your work.**

SIMPLIFYING FRACTIONS

Example 14

$$\frac{p}{2} + \frac{4p}{5}$$

(The lowest common denominator is 10)

$$= \frac{5p}{10} + \frac{8p}{10}$$

$$= \frac{13p}{10}$$

Example 15

$$= \frac{M}{4} + \frac{M+2}{3}$$

(The lowest common denominator is 12)

$$= \frac{3M}{12} + \frac{4(M+2)}{12}$$

$$= \frac{3M}{12} + \frac{4M+8}{12}$$

$$= \frac{7M+8}{12}$$

EXERCISE 5

Simplify the following expressions:

a) $\frac{r}{3} + \frac{r}{5}$

b) $\frac{w}{3} + \frac{2w}{9}$

c) $\frac{x+4}{2} + \frac{3x+1}{4}$

d) $\frac{2p+3m}{4} + \frac{2m+p}{3}$

☑ **Use the answer sheet to check your work.**

EXPONENTS (OR POWERS)

In the term $3y^2$:

- 3 is the coefficient
- y is the base
- 2 is the exponent

To simplify the equation $2y^2 + y^2$, the y^2 terms can be added together:

$$= 2y^2 + y^2$$
$$= 3y^2$$

Algebraic terms can be added or subtracted as long as their bases and exponents are the same.

Example 16 $3x^2 + 2x^2$

$= 5x^2$

Example 17 $y^3 – 3y^3 + 4y^3 + x$

$= y^3 + 4y^3 – 3y^2 + x$

$= 5y^3 – 3y^2 + x$

EXERCISE 6

Simplify the following:

a) $3m^2 - m^2$

b) $4r^2 - 2r$

c) $R^3 + 2R - P + 2R^3$

d) $3p^2q + 2qp - qp^2$

e) $6x^2y - 12x^3y + yx^2$

f) $\dfrac{2s^2 - r}{3} + \dfrac{s^2 + r}{4}$

☑ **Use the answer sheet to check your work.**

SUBSTITUTING IN FORMULAE

Substituting involves giving a letter (pronumeral) in an expression or formulae a number value so that an answer can be found.

Example 1

Find the value of $z + 4$ if $z = 3$

Using substitution $z + 4 = 3 + 4 = 7$

Example 2

To calculate the number of watts in a circuit you can use the formulae:

watts = volts x amps

W = V x A

Or W = VA

Calculate the number of watts if V = 5 and A = 3. Using substitution you get:

W = 5 x 3

W = 15

Answer: There are 15 watts in the circuit.

EXERCISE 7

Find the value of the following if W = 2 and X = 4

a) $5 + W$

b) $2W + X$

c) $6(2W - X)$

d) $\frac{X}{W}$

e) $WX-3$

f) $\frac{WX}{100}$

g) $W(2X + 5)$

h) $\frac{XW}{X + 4 + W}$

i) $\frac{3(4W + 2X)}{6W}$

j) $3W^2 - 2X + X^2W$

k) $\frac{X^2}{W}$

EXERCISE 8

a) Ohm's Law

The formula for calculating the voltage drop across a resistance when a current is flowing through it, is given by:

V = IR (answer in volts)

where V = voltage drop (volts)

I = current (amps)

R = resistance (ohms)

Find the voltage drop (V) given the following current and resistance values:

(i) I = 3A R=2Ω

(ii) I = 5A R = 1.2 Ω

(iii) I = 15 R = 0.8 Ω

b) **Electrical Power**

A formula for calculating the value of power drawn from a supply is:

$$P = \frac{V^2}{R}$$

where	P	=	power (watts)
	V	=	voltage drop (volts)
	R	=	resistance (ohms)

Find the value of power given the following voltage drop and resistance values.

(i) V = 240V R = 23Ω

(ii) V = 12V R = 24Ω

(iii) V = 49V R = 200Ω

c) Impedance

The formula for calculating the impedance in a circuit is:

$$Z = \sqrt{R^2 + X^2}$$

where Z - impedance (ohms)

R - resistance (ohms)

X - reactance (ohms)

Find the impedance of a circuit given the following values for the resistance and reactance:

(i) $R = 8\Omega$ $X = 12\Omega$

(ii) $R=3\Omega$ $X = 5.2\Omega$

(iii) $R = 42\Omega$ $X = 56\Omega$

d) Efficiency

The formula used for calculating the efficiency of an electric motor is:

$$\eta = (\frac{Pout}{Pin} x100)\%$$

where η = efficiency (%)

Pout = Power output (watts)

Pin = Power input (watts)

Calculate the efficiency of the electric motors with the following Pout and Pin values:

i) P*out* = 120W P*in* = 160W

ii) P*out* = 135W P*in* = 225W

iii) P*out* = 3000W P*in* = 3357W

☑ **Use the answer sheet to check your work.**

SOLVING EQUATIONS

An equation is a mathematical statement that two expressions are equal to each other e.g. $x + 6 = 8$

To solve an equation we find the number or numbers that make the equation true e.g. find the value of x in the above equation.

There is one basic rule in working with equations – to maintain equality you must do the same thing to each side.

Whatever is done to one side of the equation must also be done to the other side.

Example 1 $x + 6 = 8$

To solve for x it is necessary to isolate x on one side of the equation and the numbers on the other.

Step 1

$x + 6 = 8$ Subtract 6 from both sides

$x + 6 - 6 = 8 - 6$

$\mathbf{x = 2}$

Step 2

To check the answer, substitute 2 for x in the original equation.

$x + 6 = 8$

If $x = 2$

$2 + 6 = 8$

$8 = 8$

Since both sides are equal, the solution must be correct.

Example 2 $t - 3 = 10$

Step 1 Add 3 to both sides $t - 3 + 3 = 10 + 3$

Step 2 $t = 13$

EXERCISE 9

Solve the following equations:

a) $m + 8 = 18$

b) $p - 3 = 9$

c) $c - \frac{1}{2} = 2$

EXERCISE 10

Solve the following equations:

Example 3 $6x = 12$

Divide both sides by 6

$6x \div 6 = 12 \div 6$

$x = 2$

Example 4 $\frac{m}{4} = 3$

Multiply both sides by 4

$\frac{M}{4} \times 4 = 3 \times 4$

$m = 12$

a) $8y = 24$

b) $3r = 123$

c) $\frac{r}{2} = 9$

d) $\frac{y}{7} = 2$

EXERCISE 11

Solve the following equations:

Example 5 $3x + 4 = 19$

Subtract 4 from both sides

$3x + 4 - 4 = 19 - 4$

$3x = 15$

Divide both sides by 3

$3x + 3 \quad = 15 \div 3$

$x = 5$

a) $5w + 2 = 8$

b) $3p - 1 = 11$

c) $5 + 3a = 8$

d) $8t - 4 = 20$

EXERCISE 12

Solve the following equations:

Example 6 $4 (p + 3) = 20$

Divide both sides by 4

$$\frac{4(p+3)}{4} = \frac{20}{4}$$

$p + 3 = 5$

Subtract 3 from both sides

$p + 3 - 3 = 5 - 3$

$p = 2$

a) $2 (m - 4) = 14$

b) $6 (r + 8) = 120$

c) $9 (y - ½) = 36$

d) $5 (b + 12) = 65$

EXERCISE 13

Solve the following equations:

Example 7

$$\frac{q+3}{2} = 4$$

Multiply both sides by 2

$$\frac{(q+3)}{\cancel{2}} x\cancel{2} = 4x2$$

q + 3 = 8

Subtract 3 from both sides

q + 3 – 3 = 8 - 3

q = 5

Example 8

$\frac{m}{2} - 3 = 4$

Add 3 to both sides

$\frac{m}{2} - 3 + 3 = 4 + 3$

$\frac{m}{2} = 7$

Multiply both sides by 2

$\frac{m}{\cancel{2}} \times \cancel{2} = 7 \times 2$

m = 14

a) $\dfrac{p+3}{6} = 2$

b) $\dfrac{x}{5} - 4 = 6$

c) $\dfrac{r-2}{3} = 3$

d) $\dfrac{w}{4} - 1 = 8$

EXERCISE 14

Solve the following equations:

Example 9 $$\frac{9y+3}{3}=10$$

Multiply both sides by 3

$$\frac{(9y+3)}{3}x3=10x3$$ Subtract 3 from both sides

9y + 3 = 30
9y + 3 - 3 = 30 - 3
9y = 27 Divide both sides by 9
y = 3

a) $\frac{3m+1}{4}=1$

b) $\frac{2p-3}{3}=7$

c) $\frac{6(r+3)}{5}=6$

d) $\frac{2(q+2)}{4}=8$

EXERCISE 15

Solve the following:

Example 10

$3x - 3 = x + 1$

Subtract x from both sides

$3x - 3 - x = x + 1 - x$

Collect like terms

$2x - 3 = 1$

Add 3 to both sides

$2x - 3 + 3 = 1 + 3$

$2x = 4$

Divide both sides by 2

$x = 2$

Example 11

$4(s + 1) = 3(s + 2)$

Expand the brackets first

$4s + 4 = 3s + 6$

Group the like terms

$4s - 3s + 4 = 3s + 6 - 3s$

$s + 4 = 6$

$s = 6 - 4$

$s = 2$

a) $5p - 7 = 3p + 5$

b) $10m + 6 = 11m - 4$

c) $3(w - 2) = w + 2$

d) $3(4x-5)=2(5x-2)$

☑ **Use the answer sheet to check your work.**

PART B

TRANSPOSING FORMULAE

A formula is often given in such a way that to use the available information you need to rearrange or transpose the formula.

For example, the formula for calculating the number of watts in a circuit is:

W=VA

ie. **watts = volts x amps**

If you want to find the volts (V) you will need to transpose the formula to make V the subject.

ie. **V = …………**

In many ways transposing a formula is similar to solving an equation, although in this case an exact value for the letter (eg V) is not found.

Remember the rule:

Whatever is done to one side of an equation must also be done to the other side.

This rule for solving equations and the methods used in the previous section apply to **transposing** formulae.

EXERCISE 1

Transpose the following formulae to solve for the indicated letter.

Example 1

x = w - z Solve for w

Add z to both sides

x + z = w – z + z

x + z = w

Example 2

A + B = C + D Solve for A

Subtract B from both sides

A + B – B = C + D - B

A = C + D - B

a) $p = q + r$
Solve for r

b) $R_t = R_1 + R_2 + R_3$
Solve for R_2

c) $x = y - w$
Solve for w

d) $f = g + h - i$
Solve for h

e) $P_T = P_1 + P_2 + P_3$
Solve for P_3

EXERCISE 2

Transpose the formulae to solve for the indicated letter.

Example 3 s = 2r + q Solve for q

Subtract 2r from both sides

s - 2r = 2r + q - 2r

s - 2r = q

Example 4 l = 3m – n Solve for m

Add n to both sides

l + n = 3m - n + n

l + n = 3m

Divide both sides by 3

$$\frac{l+n}{3} = \frac{3m}{3}$$

$$\frac{l+n}{3} = m$$

a) a = 4b + c
Solve for c

b) z = 2w - x
Solve for w

c) s = 2r + t
Solve for r

d) f = 3g - h
Solve for h

e) $L_t = L_1 + L_2 + 2M$
Solve for M

EXERCISE 3

Transpose the following formulae to make the indicated letter the subject.

Example 5 W = VA Solve for V

Divide both sides by A

$$\frac{W}{A} = \frac{V\cancel{A}}{\cancel{A}}$$

$$\frac{W}{A} = X$$

Example 6 x = 6yw Solve for y

Divide both sides by 6w

x ÷ 6w = 6yw ÷ 6w

$$\frac{x}{6w} = \frac{6yw}{6w}$$

$$\frac{x}{6w} = y$$

a) Q = VC

Solve for C

b) ab = cd

Solve for d

c) k = 9lm

Solve for l

d) $X_L = 2\eta f L$

Solve for L

e) $P = 1^2R$

Solve for R

EXERCISE 4

Transpose the following formulae to solve for the indicated letter.

Example 7 m = $\frac{2k}{n}$ Solve for k

Multiply both sides by n

mn = $\frac{2k}{\cancel{n}}$ x $\cancel{n}$

mn = 2k

Divide both sides by 2

$\frac{mn}{2} = k$

Example 8 R = $\frac{PL}{A}$ Solve for A

Multiply both sides by A

R x A = $\frac{PL}{\cancel{A}}$ x $\cancel{A}$

RA = PL

Divide both sides by R

$\frac{\cancel{R}A}{\cancel{R}} = \frac{PL}{R}$

A = $\frac{PL}{R}$

a) $a = \frac{bc}{d}$

Solve for c

b) $f = \frac{np}{120}$

Solve for n

c) $V_e = \frac{1000C_d}{Ll}$

Solve for L

d) $P.f = \frac{R}{Z}$

Solve for *f*

EXERCISE 5

Transpose the following formulae to make the indicated letter the subject.

Example 9

$$c = \frac{3(d - e}{3}$$ solve for d

Divide both sides by 3

$$\frac{c}{3} = \frac{\cancel{3}(d - e)}{\cancel{3}}$$

$$\frac{c}{3} = d - e$$

Add e to both sides

$$\frac{c}{3} + e = d - e + e$$

$$\frac{c}{3} + e = d$$

Example 10

y = x (vw + 2) Solve for w

Divide both sides by x

$$\frac{y}{x} = \frac{\cancel{x}(vw + 2}{\cancel{x}}$$

$$\frac{y}{x} = vw + 2$$

Subtract 2 from both sides

$\frac{y}{x}$ - 2 = vw + 2 - 2

$\frac{y}{x}$ - 2 = vw

Divide both sides by v

$$\frac{y - 2x}{x} \div v = vw \div v$$

$$\frac{y - 2x}{xv} = \frac{\cancel{v}w}{\cancel{v}}$$

$$\frac{y - 2x}{xv} = w$$

a) m = 2 (In + 1)
Solve for n

b) a = bc (3d + e)
Solve for d

c) x = y (6w - z)
Solve for w

d) p=q(rs –2
Solve for

EXERCISE 6

Transpose the following formulae to solve for the indicated letter.

Example 11

$x = 2y^2$ Solve for y

Divide both sides by 2

$$\frac{x}{2} = y^2$$

Find the square root of both sides

$$\sqrt{\frac{x}{2}} = \sqrt{y^2}$$

$$\sqrt{\frac{x}{2}} = y$$

Example 12

$P = I^2R$ Solve for I

Divide both sides by R

$$\frac{P}{R} = \frac{I^2R}{R}$$

Find the square root of both sides

$$\sqrt{\frac{P}{R}} = \sqrt{I^2}$$

$$\sqrt{\frac{P}{R}} = I$$

Example 13

Q = $\sqrt{S^2 - P^2}$ Solve for S^2

Square both sides

Q = $\sqrt{S^2 - P^2}\ ^2$

Q=S-P

Add P to both sides

$Q^2 + P^2 = S^2 – P^2 + P^2$

$Q^2 + P^2 = S^2$

a) $a = b^2$

Solve for b

b) $y = 7x^2 + 3$

Solve for x

c) $x = \sqrt{\frac{y}{w}}$

Solve for w

d) $F_r = \sqrt{F_1^2 + F_2^2}$

Solve for F_2

e) $E = \frac{1}{2} mv^2$

Solve for V

☑ **Use the answer sheet to check your work.**

TRANSPOSING ELECTRICAL FORMULAE

EXERCISE 7

a) **Ohm's Law**

Ohm's law states that in any electrical circuit the current is directly proportional to the voltage and inversely proportional to the circuit resistance.

The formula for this relationship is:

$$I = \frac{V}{R}$$

Where:	I	=	current (amps)
	V	=	potential difference (volts)
	R	=	resistance (ohms)

i) What is the value of I if the potential difference is 6 volts and the resistance is 2 ohms?

ii) Transpose the formula to make V the subject and hence calculate the voltage (V) if I = 8 amps and R = 12Ω.

iii) Transpose the formula to make R the subject and hence calculate the resistance (R) if V = 11.5 volts and I = 2 amps.

b) Electrical Power

A formula for calculating the value of power in an electrical circuit is:

	P	=	I^2R
Where:	P	=	power (watts)
	I	=	current (amps)
	R	=	resistance (ohms)

i) Calculate the power in a circuit if I = 0.5 amps and R = 24 ohms.

ii) Transpose the formula to make resistance the subject and hence calculate the value of R if P = 1.5 watts and I = 0.5 amps.

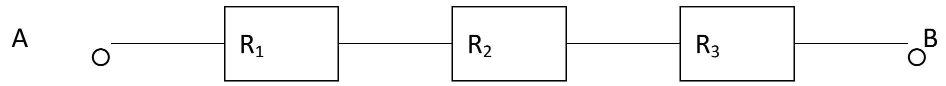
iii) Transpose the formula to make current the subject and hence calculate the value of I if R = 18 ohms and P = 4.5 watts.

c) Resistance of a Series Circuit

To find the total resistance to current flow in any series connected circuit, the values in ohms of the individual resistors are added. The formula for calculating this total resistance to current flow is:

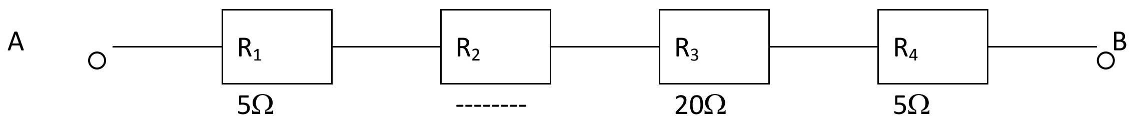
$R_{total} = R_1 + R_2 + R_3 + R_4$ .....

E.g. The diagram below represents 3 resistors in a circuit between terminal A and B.



$\therefore R_{total} = R_1 + R_2 + R_3$

i) Calculate the total resistance (R_{total}) in a series circuit with 3 resistors where $R_1 = 10\Omega$, $R_2 = 10\Omega$, and $R_3 = 5\Omega$.



ii) Find the value of R_2 in the above circuit if $R_{total} = 40\Omega$.

iii) Find the value of R_4 in a circuit with 4 resistors if $R_1 = 12\Omega$, $R_2 = 8.2\Omega$, $R_3 = 6.8\Omega$, and the total resistance is 66.

d) **A.C. Circuits**

Pythagoras' Theorem is used to calculate the resistance, reactance and impedance in AC circuits. The formula used is:

$Z^2 = R^2 + X^2$

or: $Z = \sqrt{R^2 + X^2}$

Where:

Z	=	impedance (ohms)
X	=	reactance (ohms)
R	=	resistance (ohms)

i) What is the impedance of an AC circuit with a reactance (X) of 3 ohms and a resistance (R) of 5 ohms?

ii) Transpose the formula to make R the subject and hence calculate the value of R if X = 12 ohms and Z = 24 ohms.

iii) What is the reactance of an AC circuit with an impedance of 20 ohms and a resistance of 8 ohms? (Remember. you will need to transpose the formula to make the reactance the subject.)

e) The rate of doing work (power) for a rotating body is found by using the formula:

$$P = 2\prod nT$$

Where:	P	=	power in watts
	n	=	revolutions per second (r/s)
	T	=	Torque in newton-metres (Nm)
	$2\prod$	=	6.28

i) Find the power (P) of an electric motor which is operating at 35 r/sec and has a torque of 20 Nm.

ii) Transpose the formula to make torque (T) the subject. Calculate the value of T for a compressor if the revolutions per second (n) is 20 r/s and the power (P) is 800 watts.

iii) What is the rotational speed (n) of an electric motor if the torque is 6 Nm and the power is 748 watts?

☑ **Use the answer sheet to check your work.**

ANSWERS

PART A

EXERCISE 1

a) $4A + 3A = 7A$ '
b) $3x - y$
c) $8m - 7m = m$
d) $2p + 9a + 4p - 2a = 6p + 7a$
e) $6R_1 - 4R_2 + 3R_1 = 9R_1 - 4R_2$
f) $26w - 26w = 0$
g) $4V + 6W - 2V + 8Z = 2V + 6W + 8Z$

EXERCISE 2

a) $4ab + 6ab = 10ab$
b) $xy + 5xy = 6xy$
c) $8pqr - 7pqr = pqr$
d) $5ef + 2fg - 3ef = 2ef + 2fg$
e) $15mn - 15mn = 0$
f) $8wx + 3xw = 8wx + 3wx = 11wx$
g) $9 R_1 R_2 + 7L_1L_2 - 5 R_1 R_2 + 3L_1L_2 = 4 R_1 R_2 + 10L_1L_2$
h) $2ghi + 3igh + 9hig - 2hji = 14ghi - 2hji$

EXERCISE 3

a) $p \times q \times r = pqr$
b) $3m \times n = 3mn$
c) $6h \times 51 = 30hl$
d) $2R_1 \times 4R_2 = 8R_1 R_2$
e) $5w \times 2z \times 3y = 30wzy$,

EXERCISE 4

a) $3(a + b) = 3a + 3b$
b) $5(m - n) = 5m - 5n$
c) $12(p - 2q) = 12p - 24q$
d) $4(2R_1 + 3R_2) - 3R_2$
$= 8R_1 + 12R_2 - 3R_2$
$= 8R_1 + 9R_2$
e) $3(r + s) + 4(2r - s)$
$= 3r + 3s + 8r - 4s$
$= 11r - s$
f) $6(m + 2n) + 2(4m + n)$
$= 6m + 12n + 8m + 2n$
$= 14m + 14n$

g) $8(3s + 2t) + 6(2s - 3t)$

$= 24s + 16t + 12s - 18t$

$= 36s - 2t$

h) $4(3x - y) - (x - y)$

$= 12x - 4y - x + y$

$= 11x - 3y$

i) $3r(s + 2q) + rq$

$= 3rs + 6rq + rq$

$= 3rs + 7rq$

EXERCISE 5

a) $\frac{r}{3} + \frac{r}{5}$

$= \frac{5r}{15} + \frac{3r}{15}$

$= \frac{8r}{15}$

b) $\frac{w}{3} + \frac{2w}{9}$

$= \frac{3w}{9} + \frac{2w}{9}$

$= \frac{5w}{9}$

c) $\frac{x + 4}{2} + \frac{3x + 1}{4}$

$= \frac{2(x + 4)}{4} + \frac{3x + 1}{4}$

$= \frac{2x + 8 + 3x + 1}{4}$

$= \frac{5x + 9}{4}$

d) $\frac{2p + 3m}{4} + \frac{2m + p}{3}$

$= \frac{3(2p + 3m)}{12} + \frac{4(2m + p)}{12}$

$= \frac{6p + 9m}{12} + \frac{8m + 4p}{12}$

$= \frac{10p + 17m}{12}$

EXERCISE 6

a) $3m^2 - m^2 = 2m^2$

b) $4r^2 - 2r$

c) $R^3 + 2R - P + 2R^3$

$= 3R^3 + 2R - P$

d) $3p^2q + 2qp - qp^2$

$= 2p^2q + 2qp$

e) $6x^2y - 12x^3y + yx^2$

$= 7x^2y - 12x^3y$

f) $\frac{2s^2 - r}{3} + \frac{s^2 + r}{4}$

$= \frac{4(2s^2 - r)}{12} + \frac{3(s^2 + r)}{12}$

$= \frac{8s^2 - 4r}{12} + \frac{3s^2 + 3r}{12}$

$= \frac{11s^2 - r}{12}$

EXERCISE 7

a) $5 + W = 5 + 2 = 7$

b) $2W + X$

$= 4 + 4$

$= 8$

c) $6(2W - X)$

$= 6(4 - 4)$

$= 0$

d) $X = 4 = 2$

$W = 2$

e) $WX - 3 = 8 - 3 = 5$

f) $\frac{WX}{100} = \frac{8}{100} =..\frac{2}{25}$

g) $W(2X + 5) = 2(8 + 5)$

$= 26$

h) $\frac{XW}{X+4+W} = \frac{8}{10} = \frac{4}{5}$

i) $\frac{3(4W + 2X)}{6W} = \frac{3(8 + 8)}{12} = \frac{48}{12} = 4$

j) $3W^2 - 2X + X^2W = 3 \times 4 + 2 \times 4 + 4 \times 4 \times 2$

$= 12 - 8 + 32$

$= 36$

k) $\frac{X^2}{W} = \frac{16}{2} = 8$

EXERCISE 8

a) (i) V = IR

= 3 x 2

∴ V = 6 volts

(ii) V = IR

= 5 x 1.2

∴ V = 6 volts

(iii) V = IR

= 15 x 0.8

∴ V = 12 volts

b) (i) $P = \frac{V^2}{R}$

$= \frac{(240)^2}{23}$

∴. P = 2504W

(ii) $P = \frac{V^2}{R}$

$= \frac{12^2}{24}$

∴ P = 6W

(iii) $P = \frac{V^2}{R}$

$= \frac{(49)^2}{200}$

$= \frac{2401}{200}$

∴ P = 12W

c) (i)

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{8^2 + 12^2}$$
$$\therefore Z = \sqrt{208\Omega}$$
$$= 14.4\Omega$$

(ii)

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{3^2 + (5.2)^2}$$
$$\therefore Z = \sqrt{36.04\Omega}$$
$$= 6.00\Omega$$

(iii)

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{42^2 + 56^2}$$
$$\therefore Z = \sqrt{4900\Omega}$$
$$= 70\Omega$$

d) (i)

$$\eta = \left(\frac{Pout}{Pin} x100 \right)\%$$

$$= \frac{120}{160} x100$$

$$\therefore \eta = 75\%$$

(ii)

$$\eta = \left(\frac{Pout}{Pin} x100 \right)\%$$

$$= \frac{135}{225} x100$$

$$\therefore \eta = 60\%$$

(iii)

$$\eta = \left(\frac{Pout}{Pin} x100 \right)\%$$

$$= \frac{3000}{3357} x100$$

$$\therefore \eta = 89.37\%$$

EXERCISE 9

a) m + 8 = 18
m + 8 – 8 = 18 - 8
m = 10

b) p - 3 = 9
p - 3 + 3 = 9 + 3
p = 12

c) ½ = 2
c = 2 ½

EXERCISE 10

a) 8y = 24
y = 24 ÷ 8
y = 3

b) 3r = 123
r = 41

c) $\frac{r}{2} = 9$
r = 9 x 2
r = 18

d) $\frac{y}{7} = 2$
y = 14

EXERCISE 11

a) $5w + 2 = 8$

$5w = 6$

$w = \frac{6}{5}$

b) $3p - 1 = 11$

$3p = 12$

$p = 4$

c) $5 + 3A = 8$

$3A = 3$

$A = 1$

d) $8t - 4 = 20$

$8t = 24$

$t = 3$

EXERCISE 12

a) $2(m - 4) = 14$

$\frac{2(m-4)}{2} = \frac{14}{2}$

$m - 4 = 7$

$m = 11$

b) $6(r + 8) = 120$

$\frac{6(r+8)}{6} = \frac{120}{6}$

$r + 8 = 20$

$r = 12$

c) $9(y - ½) = 36$

$\frac{9(y - ½)}{9} = \frac{36}{9}$

$y - ½ = 4$

$y - ½ + ½ = 4$

$y = 4 ½$

d) $5(b + 12) = 65$

$\frac{5(b+12)}{5} = \frac{65}{5}$

$b + 12 = 13$

$b = 1$

EXERCISE 13

a) $\frac{p+3}{6} = 2$

$p + 3 = 12$

$p = 9$

b) $\frac{x}{5} - 4 = 6$

$\frac{x}{5} - 4 + 4 = 6 + 4$

$\frac{x}{5} = 10$

$\frac{x}{5} \times 5 = 10 \times 5$

$x = 50$

c) $\frac{r-2}{3} = 3$

$\frac{r-2}{3} \times 3 = 3 \times 3$

$r - 2 = 9$

$r = 11$

d) $\frac{w}{4} - 1 = 8$

$\frac{w}{4} - 1 + 1 = 8 + 1$

$\frac{w}{4} = 9$

$\frac{w}{4} \times 4 = 9 \times 4$

$w = 36$

EXERCISE 14

a) $\frac{3m+1}{4} = 1$

$\frac{3m+1}{4} \times 4 = 1 \times 4$.

$3m + 1 = 4$

$3m + 1 - 1 = 4 - 1$

$3m = 3$

$m = 1$

b) $\frac{2p-3}{3} = 7$

$\frac{2p-3}{3} \times 3 = 7 \times 3$

$2p - 3 = 21$

$2p - 3 + 3 = 21 + 3$

$2p = 24$

$p = 12$

c) $\frac{6(r + 3)}{5} = 6$

$\frac{6(r + 3)}{5} \times 5 = 6 \times 5$

$6\ (r+3) = 30$

$\frac{6(r + 3)}{6} = \frac{30}{6}$

$r + 3 = 5$

$r = 2$

d) $\frac{2(q+2)}{4} = 8$

$\frac{2(\ q + 2)}{4} \times 4 = 8 \times 4$

$2\ (q+2) = 32$

$\frac{2\ (q+2)}{2} = 32$

$q + 2 = 16$

$q = 14$

EXERCISE 15

a) $5p - 7 = 3p + 5$

$5p - 7 - 3p = 3p + 5 - 3p$

$2p - 7 = 5$

$2p - 7 + 7 = 5 + 7$

$2p = 12$

$p = 6$

b) $10m + 6 = 11\ m - 4$

$10m + 6 - 10m = 11\ m - 4 - 10m$

$6 = m - 4$

$m - 4 = 6$

$m - 4 + 4 = 6 + 4$

$m = 10$

c) $3(w - 2) = w + 2$

$3w - 6 = w + 2$

$3w - 6 - w = w + 2 - w$

$w - 6 = 2$

$2w - 6 + 6 = 2 + 6$

$2\ w = 8$

$w = 4$

d) $3(4x - 5) = 2(5x - 2)$

$12x-15 = 10x-4$

$12x - 15 - 10x = 10x - 4 - 10x$

$2x- 15 = -4$

$2x - 15 + 15 = -4 +15$

$2\ x = 11$

$x = 5.5$

ANSWERS

PART B

EXERCISE 1

a)

$$p = q + r$$
$$p - q = q + r - q$$
$$p - q = r$$

b)

$$R_t = R_1 + R_2 + R_3$$
$$R_t - R_1 - R_3 = R_1 + R_2 + R_3 - R_1 - R_3$$
$$R_t - R_1 - R_3 = R_2$$

c)

$$x = y - w$$
$$x + w = y - w + w$$
$$x + w = y$$
$$x + w - x = y - x$$
$$w = y - x$$

d)

$$f = 9 + h - i$$
$$f - g + I = g + h - I - g + i$$
$$f - g + I = h$$

e)

$$P_T = P_1 + P_2 + P_3$$
$$P_T - P_1 - P_2 = P_1 + P_2 + P_3 - P_1 - P_2$$
$$P_T - P_1 - P_2 = P_3$$

EXERCISE 2

a)

$$a = 4b + c$$
$$a - 4b = 4b + c - 4b$$
$$a - 4b = c$$

b)

$$z = 2\,w - x$$
$$z + x = 2\,w - x + x$$
$$z + x = 2\,w$$
$$\frac{z + x}{2} = \frac{\cancel{2}\,w}{\cancel{2}}$$
$$\frac{z + x}{2} = w$$

c)

$$s = 2r + t$$
$$s - t = 2r + t - t$$
$$s - t = 2r$$
$$\frac{s - t}{2} = \frac{2r}{2}$$
$$\frac{s - t}{2} = r$$

d)

$$f = 3g - h$$
$$f + h = 3g - h + h$$
$$f + h = 3g$$
$$f + h - f = 3g - f$$
$$h = 3g - f$$

e) $L_t = L_1 + L_2 + 2M$

$L_t - L_1 - L_2 = L_1 + L_2 + 2M - L_1 - L_2$

$L_t - L_1 - L_2 = 2M$

$\frac{L_t - L_1 - L_2}{2} = \frac{\cancel{2}M}{\cancel{1}}$

$\frac{L_t - L_1 - L_2}{2} = M$

EXERCISE 3

a) $Q = VC$

$\frac{Q}{V} = \frac{\cancel{V}C}{\cancel{V}}$

$\frac{Q}{V} = C$

b) $ab = cd$

$\frac{ab}{c} = \frac{\cancel{c}d}{\cancel{c}}$

$\frac{ab}{c} = d$

c) $k = 9lm$

$\frac{k}{9m} = \frac{9lm}{9m}$

$\frac{k}{9m} = 1$

d) $X_L = 2\Pi f_L$

$\frac{X_L}{2\Pi f} = \frac{\cancel{2\Pi f}L}{\cancel{2\Pi f}}$

$\frac{X_L}{2\Pi f} = L$

e) $\frac{P}{R} = I^2$

$\frac{P}{I^2} = \frac{I^2R}{I^2}$

$\frac{P}{I^2} = R$

EXERCISE 4

a) $a = \frac{bc}{d}$

$\frac{ad}{b} = \frac{bc}{\cancel{d}} \times \cancel{d}$

$\frac{ad}{b} = \frac{\cancel{b}c}{\cancel{b}}$

$\frac{ad}{b} = c$

b) $$f = \frac{np}{120}$$

$$120\text{-}f = \frac{np}{\cancel{120}} \times \cancel{120}$$

$$\frac{120f}{p} = \frac{n\cancel{p}}{\cancel{p}'}$$

$$\frac{120f}{p} = n$$

e) $$V_c = \frac{1000V_d}{Ll}$$

$$V_cLl = \frac{1000V_d}{Ll} \times Ll$$

$$\frac{V_cLl}{V_cl} = \frac{1000V_d}{V_cl}$$

$$L = \frac{1000V_d}{V_cl}$$

d) $$P.f = \frac{R}{2}$$

$$\frac{P.f}{P} = \frac{R}{2} \times \frac{1}{P}$$

$$f = \frac{R}{2P}$$

EXERCISE 5

a) $$m = 2(ln + 1)$$

$$m = 2ln + 2$$

$$m\text{-}2 = 2ln + 2 - 2$$

$$m\text{-}2 = 2ln$$

$$\frac{m\text{-}2}{21} = \frac{2ln}{2l}$$

=> $$n = \frac{m\text{-}2}{2l}$$

b) $$a = bc\,(3d + e)$$

$$a = 3bcd + bce$$

$$a - bce = 3bcd + bce - bce$$

$$a - bce = 3bcd$$

$$\frac{a\text{-}bce}{3bc} = \frac{\cancel{3bcd}}{\cancel{3bc}}$$

=> $$d = \frac{a\text{-}bce}{3bc} \qquad \text{or} \qquad \frac{a\text{-}bce}{3bc} = d$$

c) $$x = y\,(6w - z)$$

$$\frac{x}{y} = 6w - z$$

$$\frac{x + zy}{y} = 6w$$

=> $$w = \frac{x + yz}{6y}$$

d) $p = q(rs - 2)$

$p = qrs - 2q$

$p + 2q = qrs$

$\frac{p + 2q}{qr} = \frac{qrs}{qr}$

$\Rightarrow$ $s = \frac{p + 2q}{qr}$ or $\frac{p + 2q}{qr} = s$

EXERCISE 6

a) $a = b^2$

$\sqrt{a} = \sqrt{b^2}$

$\sqrt{a} = b$

b) $y = 7x^2 + 3$

$y - 3 = 7x^2 + 3 - 3$

$y - 3 = 7x^2$

$\frac{y - 3}{7} = \frac{7x^2}{7}$

$y - 3 = x^2$

$\sqrt{\frac{y-3}{y}} = \sqrt{x^2}$

$\sqrt{\frac{y-3}{7}} = x$

c) $x = \sqrt{\frac{y}{w}}$

$x^2 = \left(\sqrt{\frac{y}{w}}\right)^2$

$x^2 = \frac{y}{w}$

$wx^2 = \frac{y}{\cancel{w}} x\cancel{w}$

$wx^2 = y$

$\frac{wx^2}{x^2} = \frac{y}{x^2}$

$w = \frac{y}{x^2}$

d)

$$F_R = \sqrt{F_1^2 + F_2^2}$$
$$F_R^2 = \left(\sqrt{F_1^2 + F_2^2}\right)^2$$
$$F_R^2 = F_1^2 + F_2^2$$
$$F_R^2 - F_1^2 = F_1^2 + F_2^2 - F_1^2$$
$$F_R^2 - F_1^2 = F_2^2$$
$$\sqrt{F_R^2 - F_1^2} = \sqrt{F_2^2}$$
$$\sqrt{F_R^2 - F_1^2} = F_2$$

e)

$$E = \tfrac{1}{2} mv^2$$
$$E \times 2 = \frac{mv^2}{2} \times 2$$
$$2E = mv^2$$
$$\frac{2E}{m} = \frac{mv^2}{m}$$
$$\frac{2E}{m} = v^2$$
$$v = \sqrt{\frac{2e}{m}}$$

EXERCISE 7

a) i)

$$I = \frac{V}{R}$$
if I = 6 and R = 2
$$I = \frac{6}{2}$$
I = 3 amps

ii)

$$I = \frac{V}{R}$$
$$IR = \frac{V}{\cancel{R}} \times \cancel{R}^1$$
IR = V
if I = 8 and R = 12
V = 8 x 12
= 96 volts

iii)

$$I = \frac{V}{R}$$
$$IR = \frac{V}{R} \times X$$
$$\frac{IR}{I} = \frac{V}{I}$$
$$R = \frac{V}{I}$$
If V = 11.5 and I = 2

$R = \frac{11.5}{2}$

$R = 5.75\Omega$

b) i) $P = 1^2R$

if I = 0.5 and R = 24

$P = (0.5)^2 \times 24$

P = 6 watts

$P = I^2R$

ii) $\frac{P}{I^2} = \frac{I^2R}{I^2}$

$\frac{P}{I^2} = R$

If P = 1.5 and I = 0.15

$R = \frac{1.5}{(0.5)^2}$

$R = 6\Omega$

$P = I^2R$

iii) $\frac{P}{R} = \frac{I^2R}{R}$

$\frac{P}{R} = I^2$

$$\sqrt{\frac{P}{R}} = \sqrt{I^2}$$

$$\sqrt{\frac{P}{R}} = I$$

If R = 18 and P = 4.5

$$I = \sqrt{\frac{4.5}{18}} = \sqrt{0.25}$$

I = 0.5amps

c) i) $R_{total} = R_1 + R_2 + R_3$

$R_{total} = 10 + 10 + 5$

$R_{total} = 25\Omega$

ii) $R_{total} = R_1 + R_2 + R_3 + R_4$

$R_{total} - R_1 - R_3 - R_4 = R_2$

If $R_{total} = 40$, $R_1 = 5$, $R_3 = 20$, $R_4 = 5$

$40 - 5 - 20 - 5 = R_2$

$R_2 = 10\Omega$

iii) $R_{total} = R_1 + R_2 + R_3 + R_4$

$R_{total} - R_1 - R_2 - R_3 = R_4$

If $R_1 = 12$, $R_2 = 8.2$, $R_3 = 6.8$, $R_{total} = 66$

$R_4 = 66 - 12 - 8.2 - 6.8$

$R_4 = 39\Omega$

d) i)

$$Z = \sqrt{R^2 + X^2}$$
$$If X = 3 and R = 5$$
$$Z = \sqrt{5^2 + 3^2}$$
$$= \sqrt{25 + 9}$$
$$= \sqrt{34}$$
$$= 5.83\Omega$$

ii)

$$Z = \sqrt{R^2 + X^2}$$
$$Z^2 = \left(\sqrt{R^2 + X^2}\right)^2$$
$$Z^2 = R^2 + X^2$$
$$Z^2 - X^2 = R^2 + X^2 - X^2$$
$$Z^2 - X^2 = R^2$$
$$\sqrt{Z^2 - X^2} = \sqrt{R^2}$$
$$\sqrt{Z^2 - X^2} = R$$
$$If X = 12 and Z = 24$$
$$R = \sqrt{24^2 - 12^2}$$
$$R = \sqrt{576 - 144}$$
$$= \sqrt{432}$$
$$R = 20.78\Omega$$

iii)

$$Z = \sqrt{R^2 + X^2}$$
$$Z^2 = \left(\sqrt{R^2 + X^2}\right)^2$$
$$Z^2 = R^2 + X^2$$
$$Z^2 - R^2 = R^2 + X^2 - R^2$$
$$Z^2 - R^2 = X^2$$
$$\sqrt{Z^2 - R^2} = \sqrt{X^2}$$
$$\sqrt{Z^2 - R^2} = X$$
$$If Z = 20 and R = 8$$
$$X = \sqrt{20^2 - 8^2}$$
$$= \sqrt{400 - 64}$$
$$= \sqrt{336}$$
$$X = 18.33\Omega$$

e) i) P = 2∏ nT
P = 6.28 x 35 x 20
P = 4396 watts

ii) $P = 2\Pi nT$

$$\frac{P}{2\Pi T} = \frac{2\Pi nT}{2\Pi n}$$

$$\frac{P}{2\Pi T} = T$$

$$ifn = 20 andP = 800$$

$$T = \frac{800}{2\Pi x 20}$$

$$= \frac{800}{6.28x20}$$

$$T = 6.37Nm$$

iii) $P = 2\Pi nT$

$$\frac{P}{2\Pi T} = \frac{2\Pi nT}{2\Pi T}$$

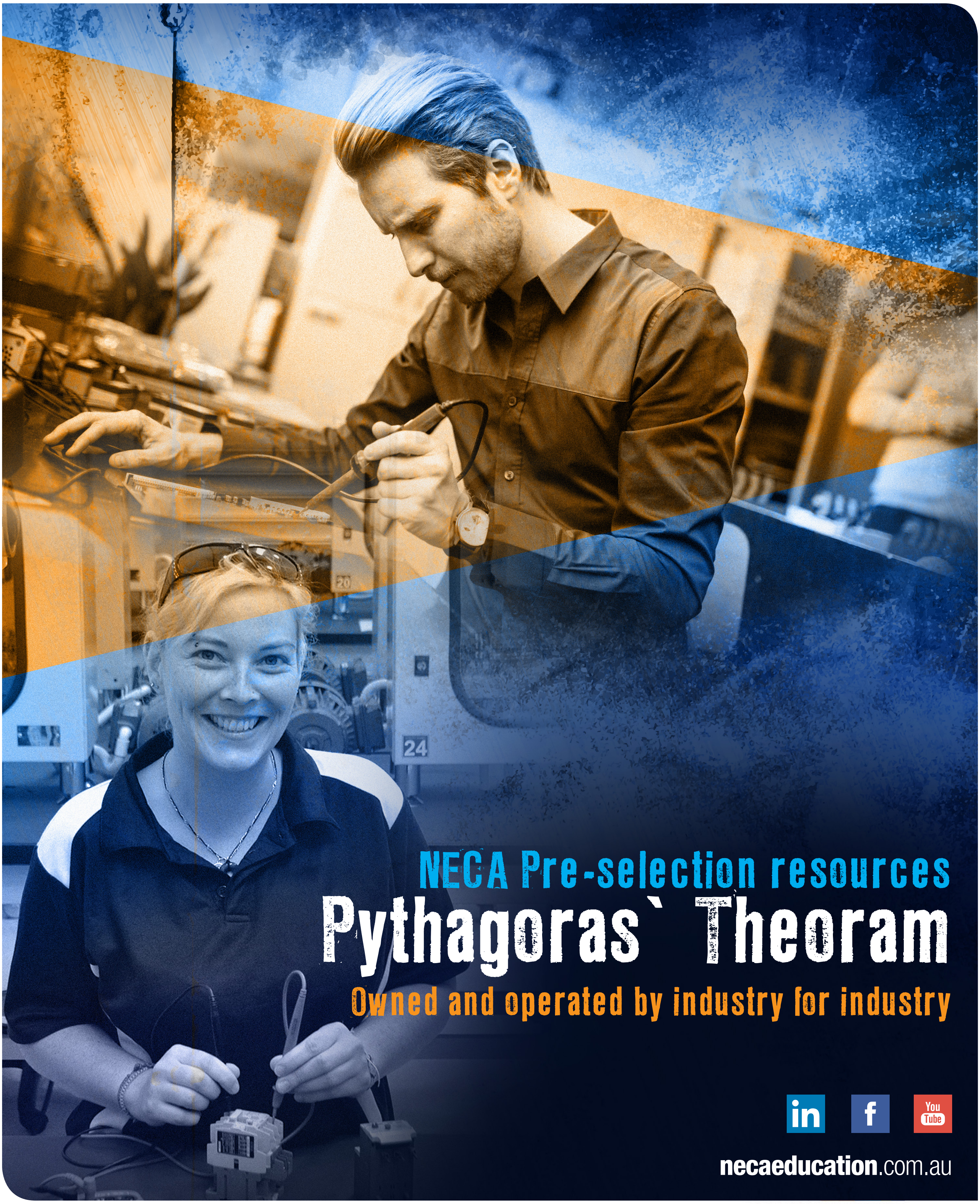
$$\frac{P}{2\Pi T} = n$$

$$IfT = 6andP = 748$$

$$n = \frac{748}{2\Pi x6}$$

$$= \frac{748}{6.28x6}$$

$$n = 19.85r/s$$



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Pythagoras` Theoram

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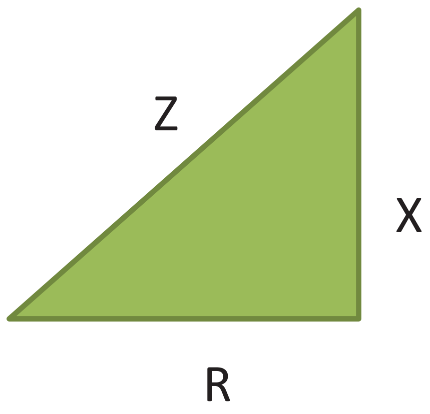
PYTHAGORGAS’ THEOREM

Pythagoras’ theorem states the relationship between the lengths of the sides of any right-angles triangle. The equation which describes this relationship ($Z^2=R^2+X^2$) is used when solving problems related to alternating current theory. For example power factor, resistance and impedance can be calculated using this Theorem.



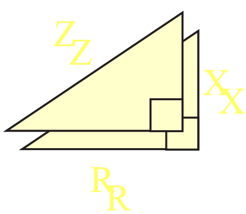
$$Z^2 = R^2 + X^2$$
$$Z = \sqrt{R^2 + X^2}$$

LEARNING OUTCOME

- Can use Pythagoras' theorem to calculate an unknown side of a right-angled triangle.

PERFORMANCE CRITERIA

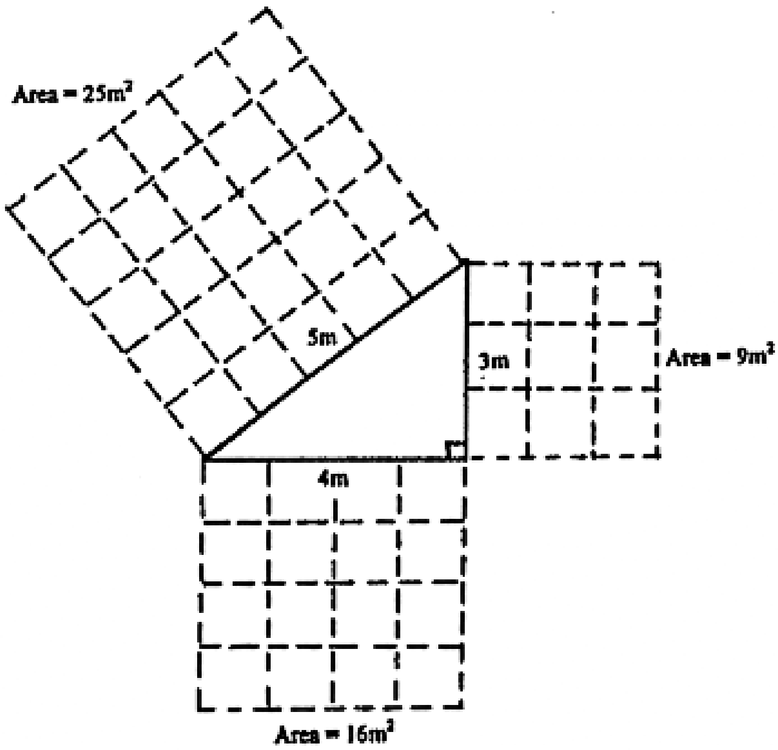
- Understands when to apply Pythagoras' theorem.
- Uses the calculator to solve problems involving Pythagoras' theorem.
- Calculates the length of an unknown side of a right-angled triangle using Pythagoras' theorem.
- Uses the calculator to solve electrical problems involving Pythagoras' theorem.



PYTHAGORAS' THEOREM

Pythagoras was a Greek mathematician who found **that in any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.**

The hypotenuse being the longest side

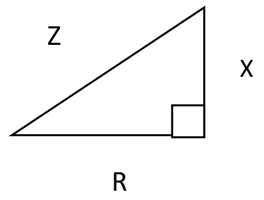


NOTE:

$5^2 = 4^2 + 3^2$

Answer: $25m^2 = 16m^2 + 9m^2$

PYTHAGORAS AND ELECTRICAL THEORY



A right-angled triangle labelled with the symbols used in electrical problems.

Z = hypotenuse
X = altitude
R = base

Pythagoras' theorem states: $Z^2 = R^2 + X^2$

therefore: $Z = \sqrt{R^2 + X^2}$

The above theorem can be used to calculate any of the three sides of a right-angled triangle when the other two sides are known.

This can be done by transposing (changing) the formula to make the subject of the equation the side you want to know.

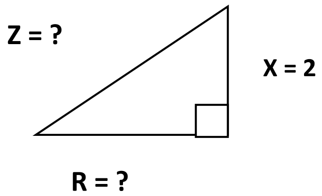
$Z^2 = R^2 + X^2$ or $Z = \sqrt{R^2 + X^2}$

$R^2 = Z^2 - X^2$ or $R = \sqrt{Z^2 - X^2}$

$X^2 = Z^2 - R^2$ or $X = \sqrt{Z^2 - R^2}$

Example 2

Find the length of the hypotenuse (Z)



$$Z = \sqrt{R^2 + X^2}$$
$$Z = \sqrt{3^2 + 2^2}$$
$$Z = \sqrt{9 + 4}$$
$$Z = \sqrt{13}$$
$$Z = 3.61$$

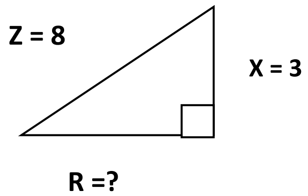
Using the calculator

Note: Steps on some calculators may differ. Refer to calculator guide.

3	**X^2**	**+**	**2**	**X^2**	**=**	**√**	**=**

Answer: 3.61

Find the length of side R.



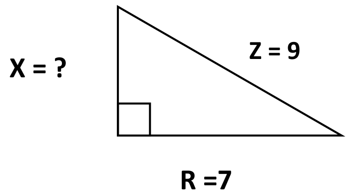
$$R = \sqrt{Z^2 - X^2}$$
$$R = \sqrt{8^2 - 3^2}$$
$$R = \sqrt{64 - 9}$$
$$R = \sqrt{55}$$
$$R = 7.42$$

Using the calculator

8	**X^2**	**-**	**3**	**X^2**	**=**	**√**	**=**

Answer: 7.42

Find the length of side X.



$$X = \sqrt{Z^2 - R^2}$$
$$X = \sqrt{9^2 - 7^2}$$
$$X = \sqrt{81 - 49}$$
$$X = \sqrt{32}$$
$$X = 5.66$$

Using the calculator

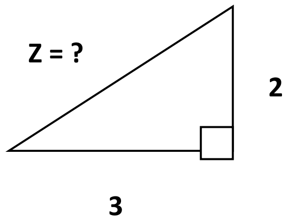
9	**X^2**	**-**	**7**	**X^2**	**=**	**√**	**=**

Answer: 5.66

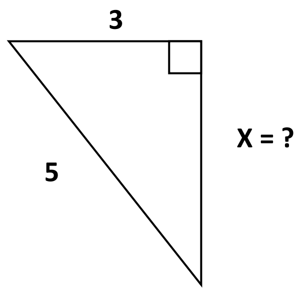
EXERCISE 1

Use Pythagoras' Theorem to find the lengths of the unknown sides of the following right-angled triangles.

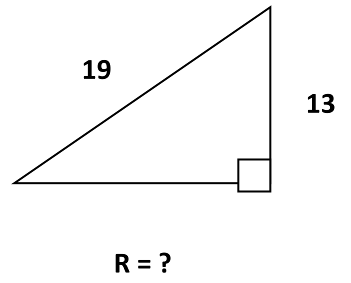
a)



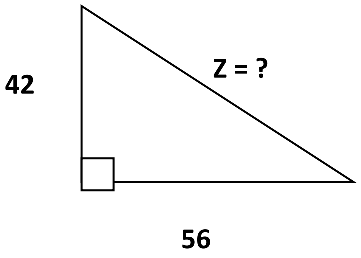
b)



c)



d)



EXERCISE 2

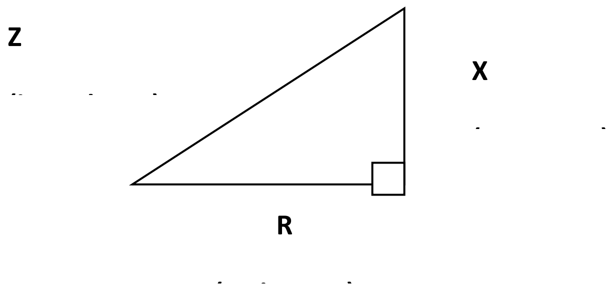
Use Pythagoras' Theorem to solve for the length of the missing sides of each triangle.

a)	X=8,	R = 13,	Z = ?
b)	Z = 20,	R = 8,	X = ?
c)	X = 12,	Z = 24,	R = ?

☑ **Use the answer sheet to check your work.**

PYTHAGORAS AND ELECTRICAL THEORY

The right angled triangle below shows the relationship between:
impedance (Z)
reactance (X)
resistance (R)



Note: Impedance, reactance and resistance are measured in ohms (Ω).

EXERCISE 3

Use the above diagram to find the impedance (Z) of a circuit with a resistance (R) of 8 ohms and reactance (X) of 12 ohms.

Answer: The impedance of the circuit is…………………………..ohms.

EXERCISE 4

Use the diagram in Exercise 3 to help solve the following problems where the values of the impedance, reactance and/or resistance are known.

a) Reactance (R) = 40 ohms

Resistance (X) = 30 ohms

Impedance (Z) = ….. ohms

b) Reactance (R) = 38.7Ω

Resistance (X) =…….Ω

Impedance (Z) = 43.6 Ω

☑ **Use the answer sheet to check your work.**

ANSWERS:

EXERCISE 1

a)

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{2^2 + 3^2}$$
$$= \sqrt{4 + 9}$$
$$Z = \sqrt{13} = 3.61$$

b)

$$X = \sqrt{Z^2 - R^2}$$
$$= \sqrt{5^2 - 3^2}$$
$$= \sqrt{25 - 9}$$
$$X = \sqrt{16} = 4$$

c)

$$R = \sqrt{Z^2 - X^2}$$
$$= \sqrt{19^2 - 13^2}$$
$$= \sqrt{361 - 169}$$
$$R = \sqrt{192} = 13.86$$

d)

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{56^2 + 42^2}$$
$$= \sqrt{3136 + 1764}$$
$$Z = \sqrt{4900} = 70$$

EXERCISE 2

a)

$$Z = \sqrt{13^2 + 8^2}$$
$$= \sqrt{169 = 64}$$
$$Z = \sqrt{233} = 15.26$$

b)

$$X = \sqrt{20^2 - 8^2}$$
$$= \sqrt{400 - 64}$$
$$X = \sqrt{336} = 18.33$$

c)

$$R = \sqrt{24^2 - 12^2}$$
$$= \sqrt{576 - 144}$$
$$= \sqrt{432} = 20.78$$

EXERCISE 3

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{8^2 + 12^2}$$
$$= \sqrt{64 + 144}$$
$$Z = \sqrt{208} = 14.42\Omega$$

EXERCISE 4

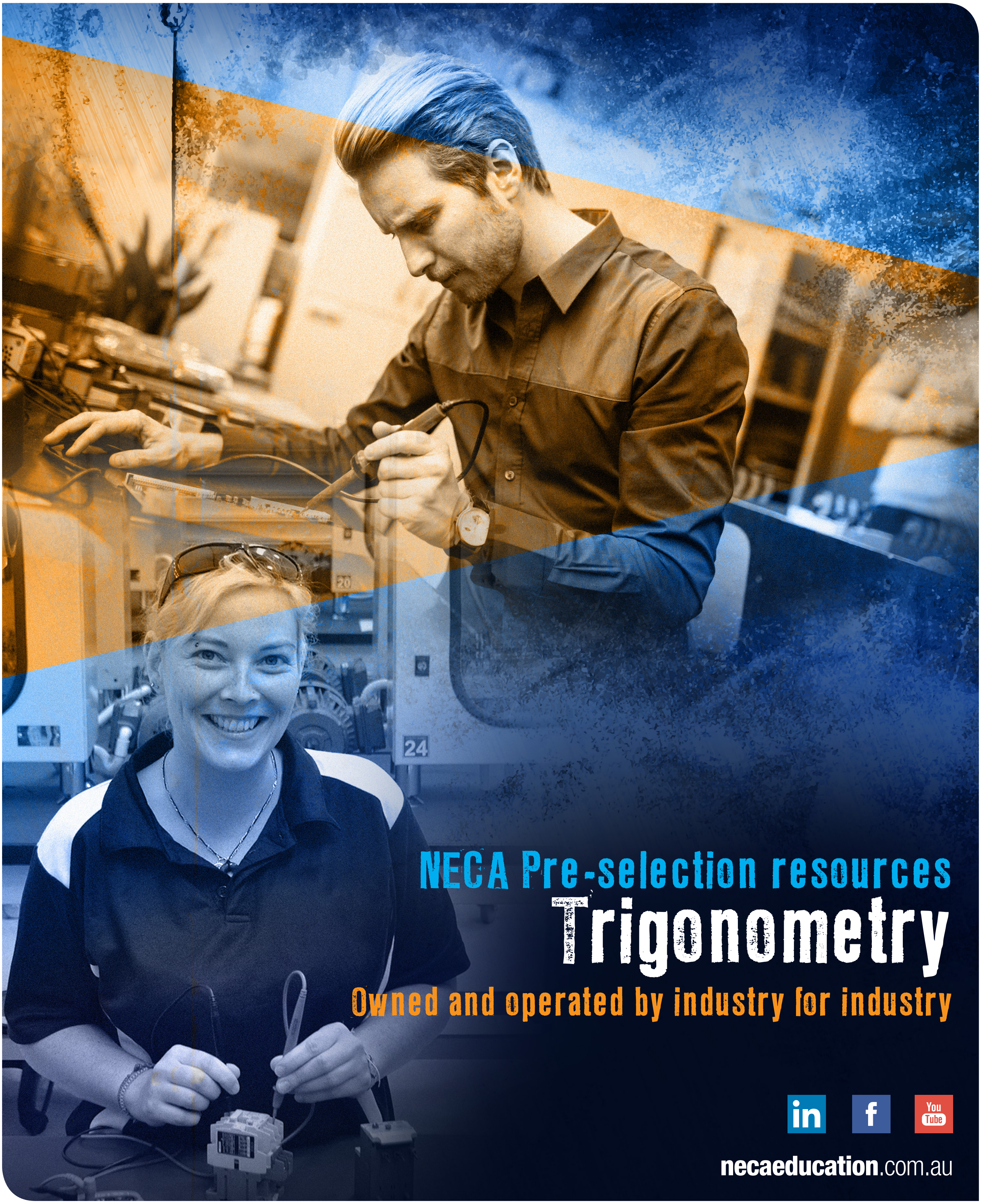
a)

$$Z = \sqrt{40^2 + 30^2}$$
$$= \sqrt{1600 + 900}$$
$$Z = \sqrt{2500} = 50\Omega$$

b)

$$X = \sqrt{43.6^2 - 38.7^2}$$
$$= \sqrt{1900.96 - 1497.69}$$
$$X = \sqrt{403.27} = 20.08\Omega$$

neca Education & Careers



NECA Pre-selection resources

Trigonometry

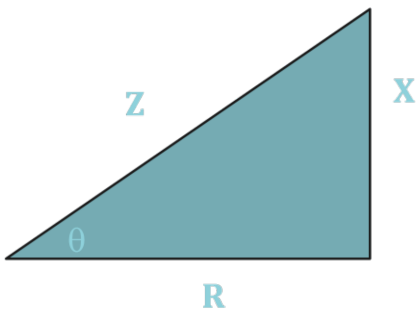
Owned and operated by industry for industry

necaeducation.com.au

TRIGONOMETRY

Trigonometry is a means of solving for sides and angles in a right-angled triangle. This information can then be used to calculate the magnitude and direction of forces in the electrical trade.

For example, in circuits which contain inductance and capacitance trigonometry can be used to determine the relationship between the current and voltage. The functions are also used to calculate phase angles.



LEARNING OUTCOME

- Can use the trigonometric functions to calculate an unknown side/s and angle/s in a right-angled triangle.

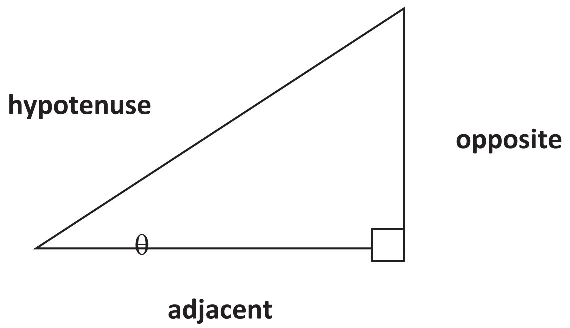
PERFORMANCE CRITERIA

- Uses the scientific calculator to find the sine, cosine or tangent of an angle.
- Identifies when to use the sin, cos or tan ratio to find an unknown side or angle in a right-angled triangle.
- Transposes the trigonometric ratios in order to solve for sides or angles.
- Uses the scientific calculator to solve worded problems involving the trigonometric functions.

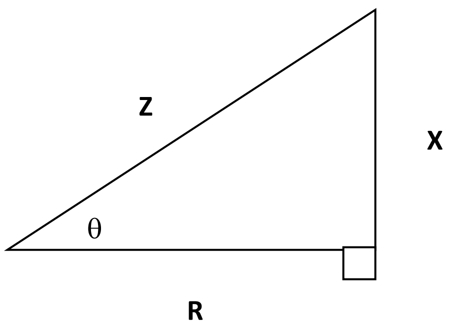
TRIGONOMETRIC FUNCTIONS

Trigonometric functions provide information about the size of the angles in right angled triangles. These functions express the ratio between any two sides of a right triangle.

When an angle is given (the Greek letter θ is used as a general angle) then the sides of the triangle are labelled with respect to this angle (θ).



1. The longest side is the hypotenuse.
2. The side opposite the angle is the opposite side.
3. The side next to the angle is the adjacent side.



3 functions expressing a ratio of the length of one side to another:

1. Sine of θ = $\frac{\text{opposite side}}{\text{hypotenuse}}$

   or

   $\sin\theta = \frac{opp}{hyp} = \frac{X}{Z}$

2. Cosine of θ = $\frac{\text{adjacent side}}{\text{hypotenuse}}$

   or

   $\cos\theta = \frac{adj}{hyp} = \frac{R}{Z}$

3. Tangent of θ = $\frac{\text{opposite side}}{\text{adjacent side}}$

   or $\tan\theta = \frac{opp}{adj} = \frac{X}{R}$

TRIGONOMETRIC FUNCTIONS – USING THE CALCULATOR

A scientific calculator can be used to obtain the sine, cosine or tangent of an angle. The answer is expressed in radians.

Note: The following steps are used on a Sharp calculator, other calculators may use different steps. Read your operators manual to determine the steps used on your calculator.

ANGLES LESS THAN 90°

Example 1

Using the Microsoft calculator:

Problem	Keysteps					Answer
sin 18°		1	8	sin		0.3090
cos 40°		4	0	cos		0.7660
tan 70°		7	0	tan		2.7474
cos 75.8	7	5	.	8	cos	0.2453
sin 14.2	1	4	.	2	sin	0.2453

Example 2

Finding the sine, cosine and tangent of angles expressed in degrees and minutes using the D°M'S key (degrees, minutes, seconds) of some calculators:

Problem	Keysteps							Answer
cos 26°54’	cos	2	6	D°M’S	5	4	=	0.8918
sin 60°12’	sin	6	0	D°M’S	1	2	=	0.8678

EXERCISE 1

Use the calculator to find the values of the following. Give answers correct to 4 decimal places.

a. sin 30°

b. cos 65°

c. tan 20°

d. sin 71 °13'

e. cos 8°19'

f. tan 23.5°

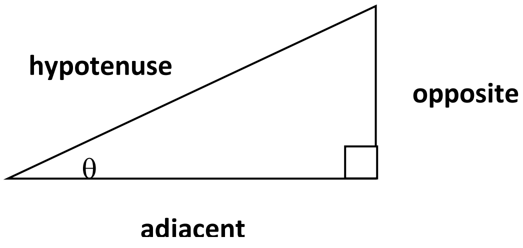
g. sin 79° 3'

h. cos 81°45'

☑ **Use the answer sheet to check your work**

FINDING THE UNKNOWN SIDES

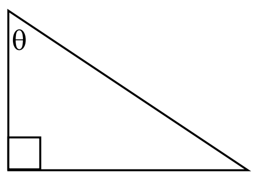
To use the trigonometric ratios to find unknown sides, it is important to be clear about which side is the opposite, adjacent and hypotenuse in any given triangle.



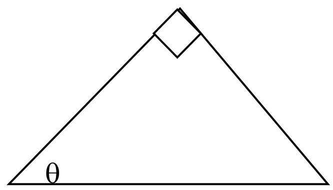
EXERCISE 2

Label the sides of each of the triangles below to show which sides are the opposite, adjacent or hypotenuse.

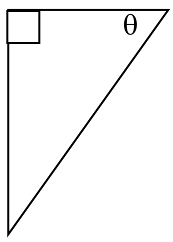
a.



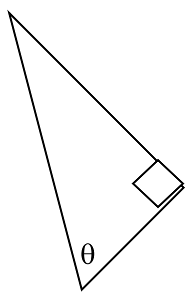
b.



c.

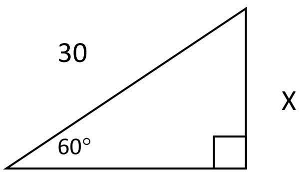


d.



Example 3

Use the sin ratio to find the side marked x.



Remember: $\sin\theta = \dfrac{opp}{hyp}$

$\sin 60° = \dfrac{X}{30}$

$\therefore X = 30 \times \sin 60°$

$= 30 \times 0.8660$

$= 25.9808$

$= 25.98$

Using the Microsoft calculator:

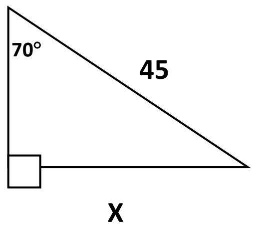
3	**0**	**x**	**6**	**0**	**sin**

Answer: 25.9808
Rounded: 25.98

EXERCISE 3

Use the sin ratio to find the side marked X.

a.



sin 70° = $\frac{X}{45}$

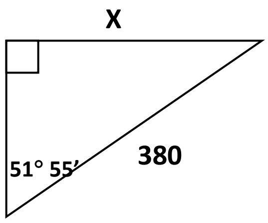
∴ X = 45 x sin 70°

=

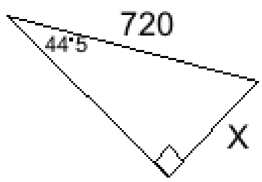
=

=

b.



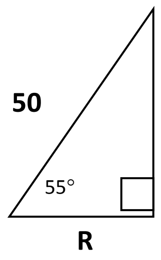
c.



Example 4

Use the cos ratio to find the side marked R. Give answer correct to 2 decimal places.

Remember: Cos $\theta = \dfrac{adj}{hyp}$



$\cos 55° = \dfrac{R}{50}$

$\therefore \quad R = 50 \times \cos 55°$

$= 50 \times 0.5736$

$= 28.6788$

≈ 28.68


Using the calculator

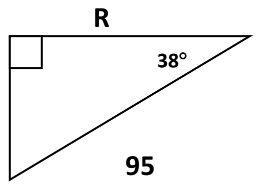
5	0	x	5	5	cos

Answer: 28.6788
Rounded: 28.68

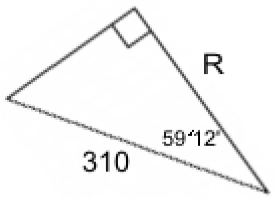
EXERCISE 4

Use the cos ratio to find the side marked R. Give answers correct to 2 decimal places.

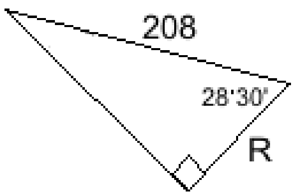
a.



b.

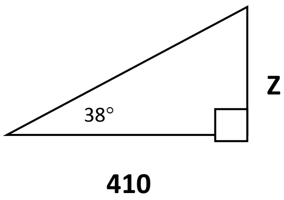


c.



Example 4

Use the tan ratio to find the side marked Z.



$\tan 38° = \dfrac{Z}{410}$

$\therefore Z = 410 \times \tan 38°$

$= 410 \times 0.7813$

$= 320.3271$

$= 320.33$

Using the calculator

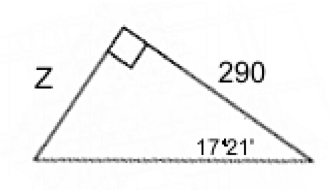
4	**1**	**0**	**x**	**3**	**8**	**tan**

Answer: 320.271
Rounded: 320.33

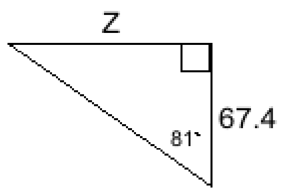
EXERCISE 5

Use the tan ratio to find the side marked Z.

a.



b.



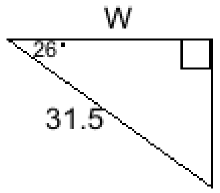
☑ **Use the answer sheet to check your work.**

EXERCISE 6

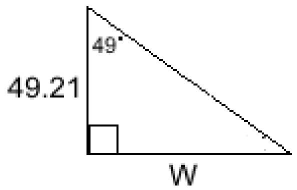
Find the unknown sides (W) in the following triangles. You will need to decide which ratio to use for each of the triangles.

ie. sin θ = opp / hyp   cos θ = adj / hyp   or   tan θ = opp / adj

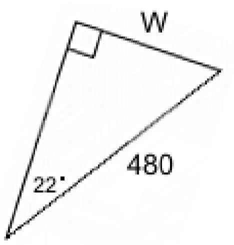
a.



b.



c.

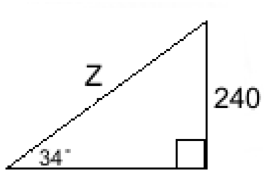


Use the answer sheet to check your work.

TRANSPOSING THE RATIOS

Example 5

Find the unknown side (Z).



$\sin 34° = \dfrac{240}{Z}$ (Multiply both sides by Z)

$Z \times \sin 34° = 240$ (Divide both sides by sin34°)

$Z = \dfrac{240}{\sin 34°}$

$Z = \dfrac{240}{0.5592}$

. . $Z = 429.19$

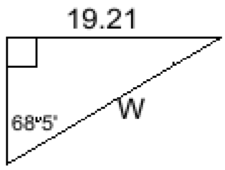

Using the calculator

2	4	0	÷	3	4	sin	=	Answer: 429.19

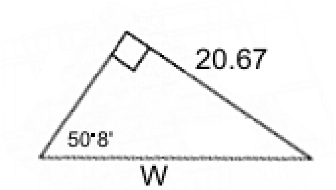
EXERCISE 7

Find the unknown sides (W) in the following triangles. Give answers correct to 2 decimal places.

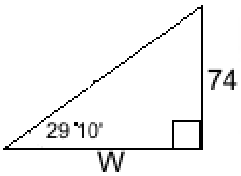
a.



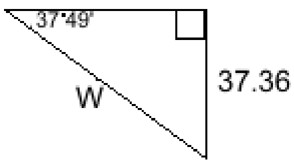
b.



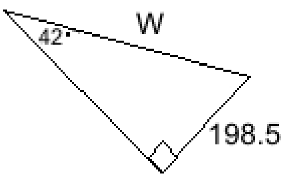
c.



d.



e.



☑ **Use the answer sheet to check your work.**

FINDING ANGLES

The three trigonometric ratios can be used to find an angle when two or three sides in the triangle are known.

The first step in such a calculation is to determine which ratio needs to be used. Then the inverse keys on the scientific calculator can be used to calculate the size of the angle.

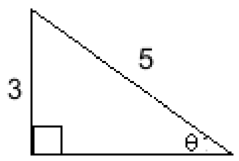
The **inverse** trigonometric keys (or arc functions) are:

$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

eg. $\sin^{-1}$ 0.45 means "the sine who's angle is 0.45"
$\cos^{-1}$ 0.6 means "the cosine who's angle is 0.6"

Example 6

Find the angle θ



The opposite side and hypotenuse are given $\therefore$ the sin ratio can be used to find θ.

$$\text{Sin } \theta = \frac{\text{opposite}}{\text{Hypotenuse}}$$

$$\sin \theta = \frac{3}{5}$$

$$= 0.6$$

$$\sin^{-1} 0.6 = 36°52'$$

3	÷	5	=	2^{nd} F	$\sin^{-1}$	=	D°M'S	Answer: 36°52'

EXERCISE 8

Use the arc functions (inverse trig) keys to find the following. Express the answers in degrees and minutes.

a. $\sin^{-1} 0.7705$

2^{nd} F	**$\sin^{-1}$**	**.**	**7**	**7**	**0**	**5**	**=**

b. $\sin^{-1} 1$

c. $\cos^{-1} 1$

d. $\cos^{-1} 0.9655$

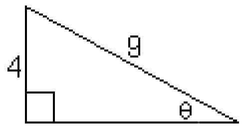
e. $\tan^{-1} 0.2793$

f. $\cos^{-1} 0.2924$

EXERCISE 9

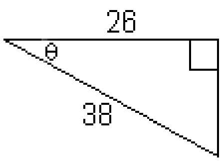
Find the unknown angle θ in the following triangles. You will need to first of all decide which ratio to use and then use the appropriate inverse key.

a.

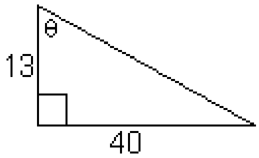


(Opposite and hypotenuse are known
∴ use $\sin^{-1}$)

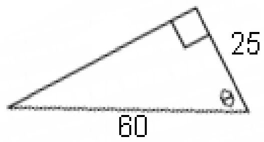
b.



c.



d.



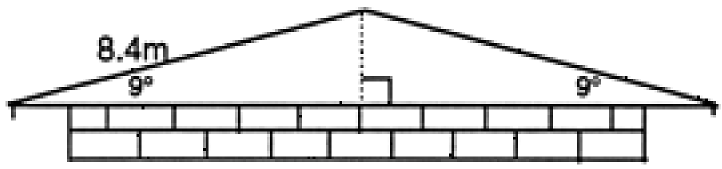
e. In a right angled triangle θ = 54° and the hypotenuse is 10mm in length. Find the lengths of the opposite and adjacent sides.

f. Angle θ in a right angled triangle is 24° and the opposite side is 13 units in length. Solve for the lengths of the other two sides.

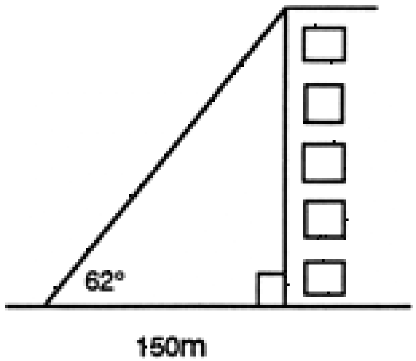
PRACTICAL APPLICATIONS

EXERCISE 10

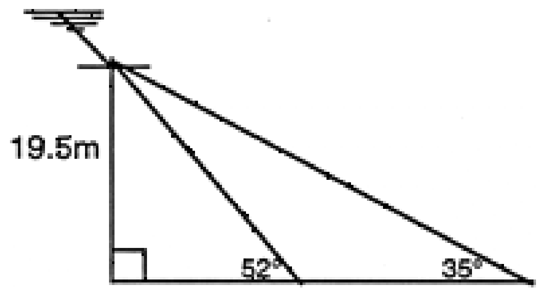
a. The diagram below 'represents the cross-section of a roof of a house. If the pitch of the roof is 9°, how high is the top of the roof above the ceiling?



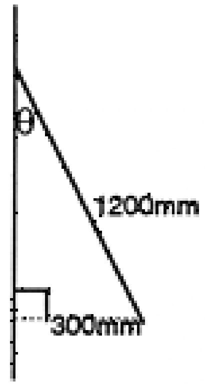
b. An electrician needs to run some conduit from the base of a tall building to the top. He uses the measurements shown below to calculate the required length of conduit.
How much conduit is needed?



c. Two cables are used to secure a TV antenna 19.5 metres high. Calculate the length of each cable given the angles shown in the diagram.



d. Angle detecting switches for burglar alarms are rated to activate at various angles. If a 1200mm long window opens 300mm at the base, specify the angle of the switch you would need.

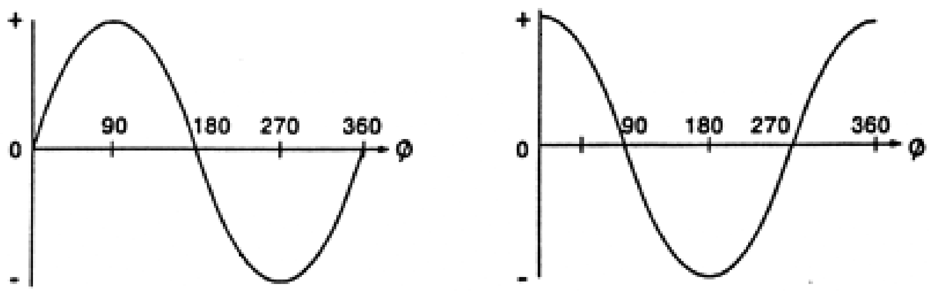


ANGLES GREATER THAN 90°

The value of the sin, cos and tan ratios are either negative or positive depending on the size of the angle. The diagram below summarises the signs of the trigonometric functions.

sin+ cos- tan- Quadrant 2	sin+ cos+ tan+ Quadrant 1
sin- cos- tan+ Quadrant 3	sin- cos+ tan- Quadrant 4

The features of the sine and cosine function are shown below highlighting the relationship between angle size and positive and negative values.



(a) the sine function (b) the cosine function

The sine and cosine functions are the main functions used when studying AC voltages and currents.

EXERCISE 11

Use the calculator to compare the values of the following functions.

a. sin 30°

b. sin 210°

c. cos 184°

d. cos 4°

e. sin 97°

f. sin 83°

g. cos 20°

h. cos 340°

i. sin 330°

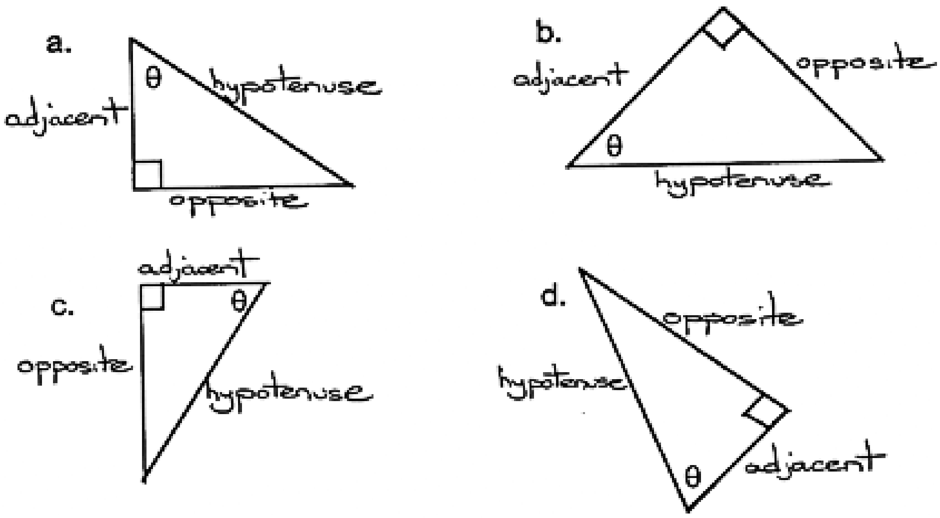
j. cos 300°

ANSWERS

EXERCISE 1

a. 0.5000
b. 0.4226
c. 0.3640
d. 0.9467
e. 0.9896
f. 0.4348
g. 0.9818
h. 0.1435

EXERCISE 2



EXERCISE 3

a. $\sin 70° = \frac{X}{45}$

$\therefore X = 45 \times \sin 70°$
$= 45 \times 0.9397$
$= 42.2862$
$= 42.29$

b. $\sin 55°55' = \frac{X}{380}$

$X = 299.10$

c. $\sin 44°5' = \frac{X...}{720}$

$X = 500.91$

EXERCISE 4

a.	Cos 38°	=	$\frac{R}{95}$
	∴ R	=	95 x cos 38°
		=	95 x 0.7880
		=	74.8610
		=	74.86
b.	cos 59°12'	=	$\frac{R}{310}$
	R	=	158.73
c.	cos 28°30'	=	$\frac{R}{208}$
	R	=	182.79

EXERCISE 5

a.	tan 17021'	=	$\frac{Z}{290}$
	∴ Z	=	290 x tan 17°21'
		=	290 x 0.3124
		=	90.6026
		=	90.60
b.	tan 81°	=	$\frac{Z}{67.4}$
	Z	=	425.55

EXERCISE 6

a.	cos 26°	=	$\frac{W}{31.5}$
	∴ W	=	31.5 x cos 26°
		=	31.5 x 0.8988
		=	28.3120
		=	28.31
b.	tan 49°	=	$\frac{W}{49.21}$
	∴ W	=	56.61
c.	sin 22°	=	$\frac{W}{480}$
	W	=	179.81

EXERCISE 7

a.	sin 68°5'	=	<u>19.21</u>

W

$W = 20.71$

b. $\sin 50°8' = \frac{20.67}{W}$

$W = 26.93$

c. $\tan 29°10' = \frac{74}{W}$

$W = 132.59$

d. $\sin 37°49' = \frac{37.36}{W}$

$W = 60.93$

e. $\sin 42° = \frac{198.5}{W}$

$W = 296.65$

EXERCISE 8

a. 50°24'
b. 90°
c. 0
d. 15°6'
e. 15°36'
f. 73°

EXERCISE 9

a. $\sin\theta = \frac{4}{9}$

$\sin 26°23' = \frac{4}{9}$

b. $\cos\theta = \frac{26}{38} = \frac{13}{19}$

$\cos 46°50' = \frac{13}{19}$

c. $\tan\theta = \frac{40}{13}$

$\tan 72° - \frac{40}{13}$

d. $\cos\theta = \frac{25}{60} = \frac{5}{12}$

$\cos 65°23' = \frac{5}{12}$

e.

Opposite side	Adjacent Side
$\sin\theta = \frac{X}{10}$	$\cos\theta = \frac{R}{10}$
X = 8.09mm	R = 5.88mm

f.

Hypotenuse (Z)	Adjacent (R)
$\sin\theta = \frac{13}{Z}$	$\tan\theta = \frac{13}{R}$
Z = 31.96 units	R = 29.20 units